

6

Capacitance

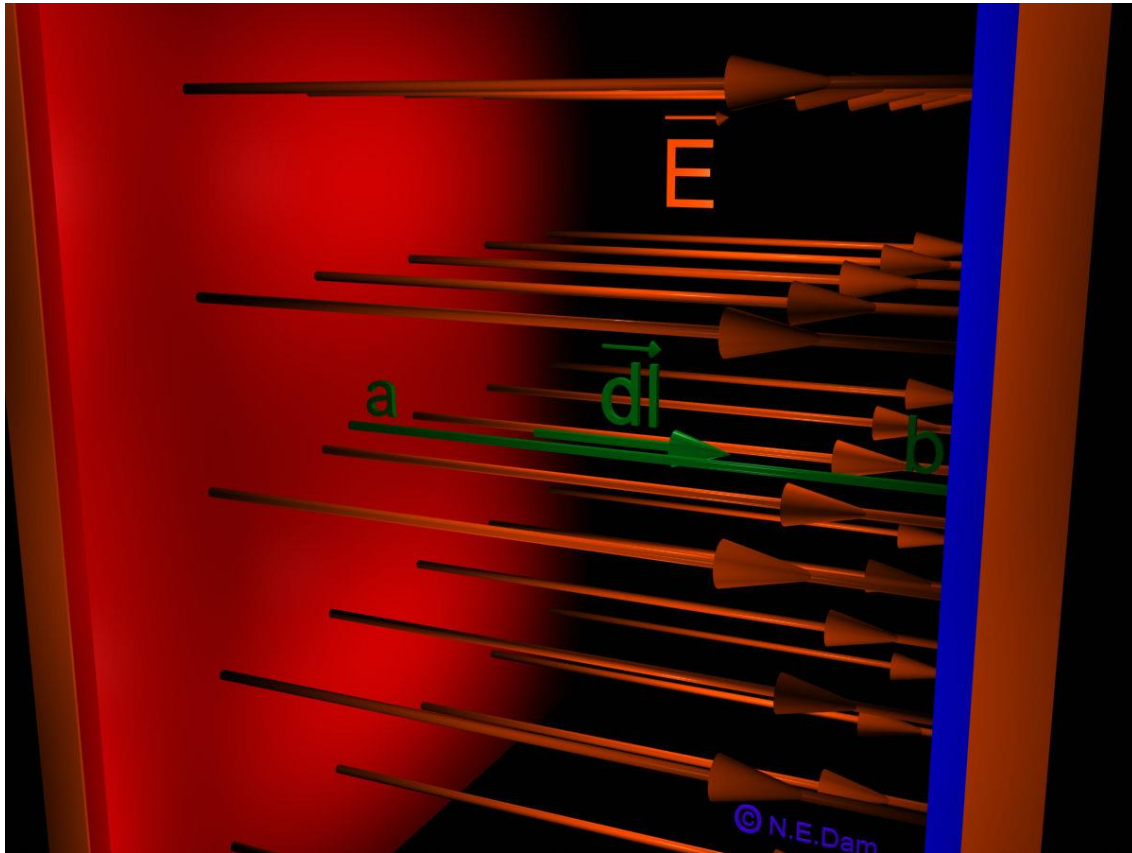


Fig.6.1.1, ref.1 (N.E. Dam ©). A capacitor consists of two conductors (a and b) separated by an insulator, which may be vacuum. The two conductors are often realized as two metal plates with positive and negative charges Q and $-Q$. The E -field is from the positively charged plate (a) to the negatively charged plate (b). This E -field may be considered as being uniform (constant) if the two plates are relatively closely spaced.

Copyright © 2018 Niels Ebbe Dam (N.E.Dam). This version is revised 20190310.

All rights reserved. The text in this chapter is by N.E.Dam and is protected by copyright. All figures in this chapter marked by ref.1 (N.E. Dam ©) are by N.E.Dam and are protected by copyright. N.E. Dam is an associate professor at DTU Ballerup Campus, Denmark.

6.1 Introduction

Capacitors are basically charge storage devices. They are very important in many applications and in electronic equipment: photo flash, pulsed lasers, air bag sensors, radio and TV, circuits with filters and time delays, power supplies etc.

Fig.6.1.2 shows a capacitor for demonstration purposes, where the plate area and the distance between the plates can be measured and thus calculate the expected capacitance, which may be compared with the measured capacitance.

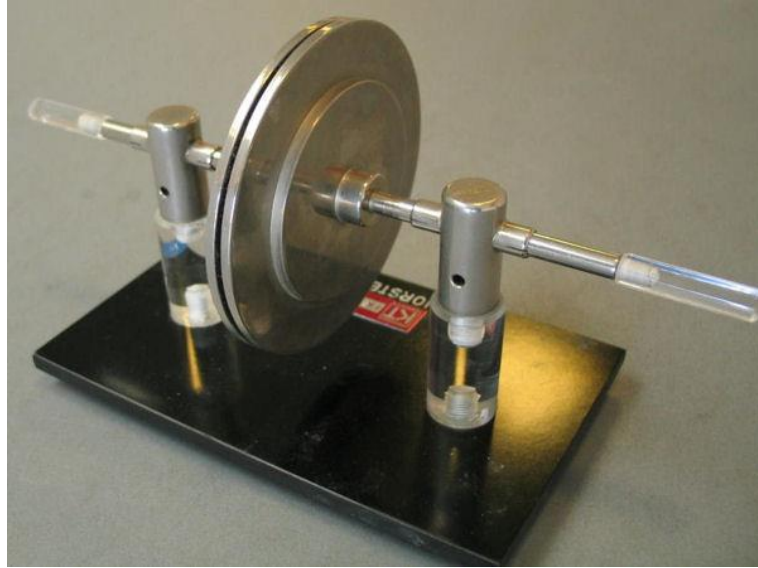


Fig.6.1.2, ref.1 (N.E. Dam ©). A capacitor for demonstrations.

6.2 Definition of capacitance

A capacitor consists of two conductors (a and b) separated by an insulator (the insulator may be vacuum) as shown in fig.6.2.1 and fig.6.2.2. The capacitor is charged when electrons are transferred from a to b through an external circuit.

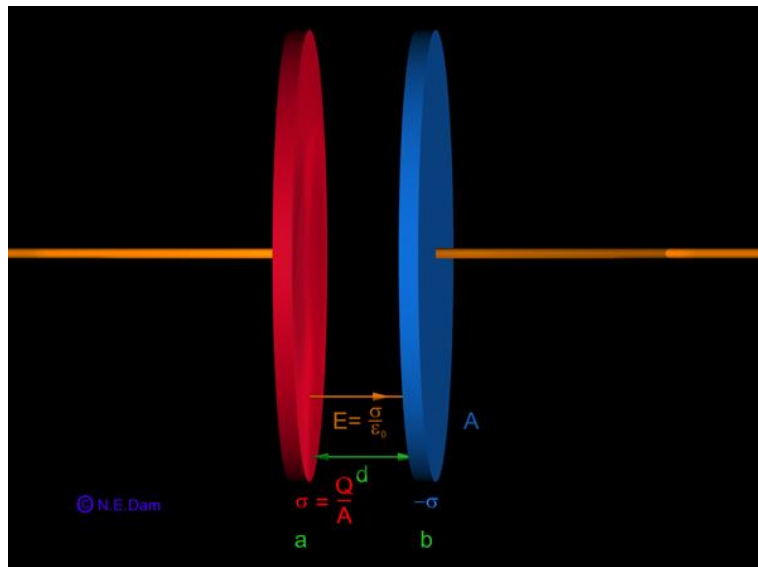


Fig.6.2.1, ref.1 (N.E. Dam ©). Plate capacitor.

The two capacitor plates are usually relatively close to each other and they may be considered as a double layer of charge, where the E-field is constant. In chapter 3, the section “Field inside a double layer of charge”, we found a result for the E-field in-between the two layers:

$$E = \frac{\sigma}{\epsilon_0} \quad \text{eq. 6.2.1}$$

In chapter 5, the section “Basic definition of electric potential” we found, that the E-field inside a double layer of charge (where the E-field is constant) is:

$$E = \frac{V_{ab}}{d} \quad \text{eq. 6.2.2}$$

where V_{ab} is the potential difference between the two capacitor plates and d is the distance between the two plates as illustrated in fig.6.2.2.

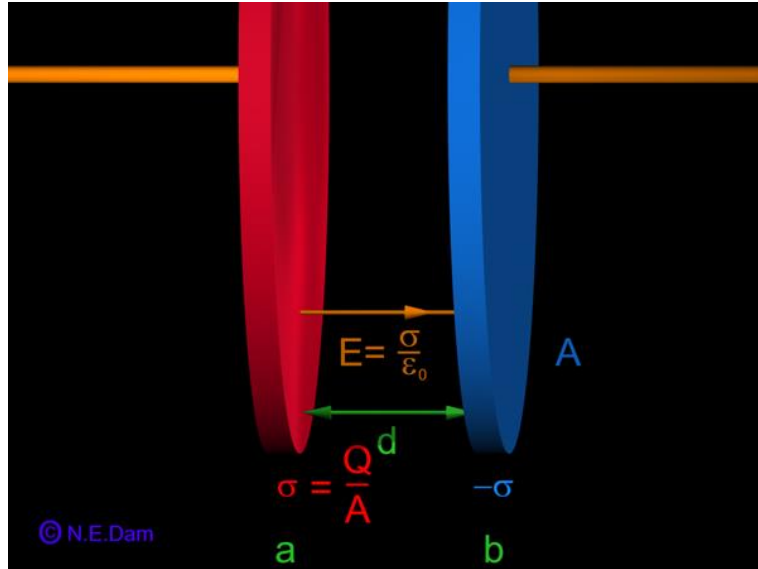


Fig.6.2.2, ref.1 (N.E. Dam ©). Close-up of the above plate capacitor.

The potential difference is therefore

$$V_{ab} = V_a - V_b = Ed = \frac{\sigma d}{\epsilon_0} = \frac{Qd}{A\epsilon_0} \quad \text{eq. 6.2.3}$$

Here ϵ_0 is the so-called vacuum permittivity

$$\epsilon_0 = 8,854 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}} \quad \text{eq. 6.2.4}$$

The capacitance is defined as

$$C \equiv \frac{Q}{V_{ab}} \quad \text{eq. 6.2.5}$$

According to eq.6.2.1, V_{ab} is proportional to Q . Hence the capacitance must be a constant ratio between the charge and the potential difference.

The unit is $\text{C} / \text{V} = \text{F}$ (Farad).

6.3 Plate capacitor

A plate capacitor may be rectangular or circular as in fig.6.2.1. According to eq.6.2.5 and eq.6.2.3, the capacitance is

$$C \equiv \frac{Q}{V_{ab}} = Q \frac{A\epsilon_0}{Qd} = \epsilon_0 \frac{A}{d} \quad \text{eq. 6.3.1}$$

The capacitance is proportional with the plate area and inverse proportional with the distance between the two plates.

6.4 Spherical capacitor

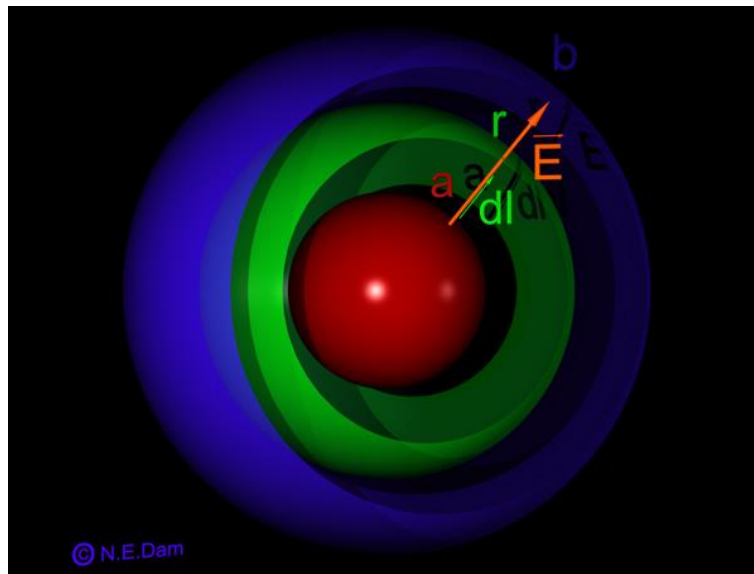


Fig.6.4.1, ref.1 (N.E. Dam ©). A spherical capacitor.

We want to find the capacitance C for the spherical capacitor shown in fig.6.4.1. First we express V_{ab} by means of the charge Q on the inner sphere. First we place a Gaussian surface, a sphere of radius r , and use Gauss's law to find the E -field:

$$\Phi_E = \oint_{GS} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \Rightarrow E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} \quad \text{eq. 6.4.1}$$

According to chapter 5 we have

$$V_{ab} = \frac{W_{ab}}{q_0} = \int_a^b \vec{E} \cdot d\vec{l}$$

Because $\vec{E} \parallel d\vec{l}$ we remove the vector signs. Then we use eq.6.4.1 to get

$$V_{ab} = \int_a^b E \cdot dl = \int_a^b E \cdot dr \Rightarrow$$

$$V_{ab} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q}{4\pi\epsilon_0} \frac{b-a}{ab} \quad \text{eq. 6.4.2}$$

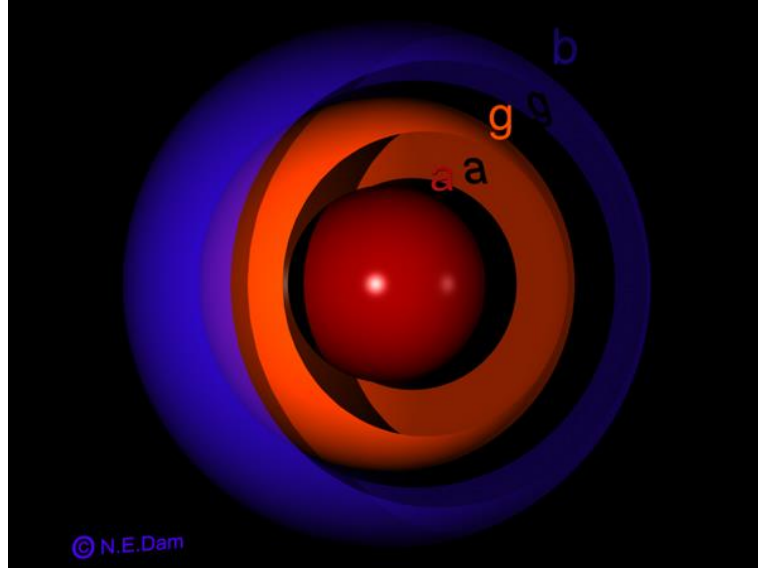


Fig.6.4.2, ref.1 (N.E. Dam ©). A spherical capacitor with the geometric mean area shown as the orange sphere.

The reason why we write $\left(\frac{1}{a} - \frac{1}{b}\right)$ as $\frac{b-a}{ab}$ with common denominator is that it is easier to take the reciprocal of $\frac{b-a}{ab}$ rather than $\left(\frac{1}{a} - \frac{1}{b}\right)$. This is what is done below when we find the capacitance of the spherical capacitor.

$$C = \frac{Q}{V_{ab}} = Q \frac{4\pi\epsilon_0}{Q} \frac{ab}{b-a} = \epsilon_0 \frac{4\pi ab}{b-a} = \epsilon_0 \frac{A}{d} \quad \text{eq. 6.4.3}$$

If we compare eq.6.4.3 with the plate capacitor eq.6.3.1, we see that the capacitance of the spherical capacitor is the same as for the plate capacitor, if we use $A=4\pi ab$ for the area and $d=b-a$ for the distance between the plates.

In math we have more types of average values.

If we have n numbers $a_1, a_2, \dots, a_n$ we may calculate

The usual average value is

$$m = \frac{1}{n} \{a_1 + a_2 + \dots + a_n\} = \frac{1}{n} \sum_{i=1}^{i=n} a_i \quad \text{eq. 6.4.4}$$

However, it is also possible to calculate the so-called geometric average value:

$$g = \{a_1 \cdot a_2 \cdot \dots \cdot a_n\}^{\frac{1}{n}} = \left\{ \prod_{i=1}^{i=n} a_i \right\}^{\frac{1}{n}} \quad \text{eq. 6.4.5}$$

In the case where we have only two numbers $a_1=a$ and $a_2=b$ we get $g=(ab)^{1/2}$.

$$g = \{a \cdot b\}^{\frac{1}{2}} \Rightarrow g^2 = ab \quad \text{eq. 6.4.6}$$

The area of the outer spherical capacitor plate is $4\pi b^2$.

The area of the inner spherical capacitor plate is $4\pi a^2$.

In between those two plates we have a sphere (shown as orange in fig.6.4.2) where the area is $4\pi g^2 = 4\pi ab$ which is the geometric mean between the two capacitor plates.

If we use this area in the formula for the capacitance of a plate capacitor and if we use the physical distance d between the two plates, we get the capacitance of the spherical capacitor.

6.5 Different types of capacitors

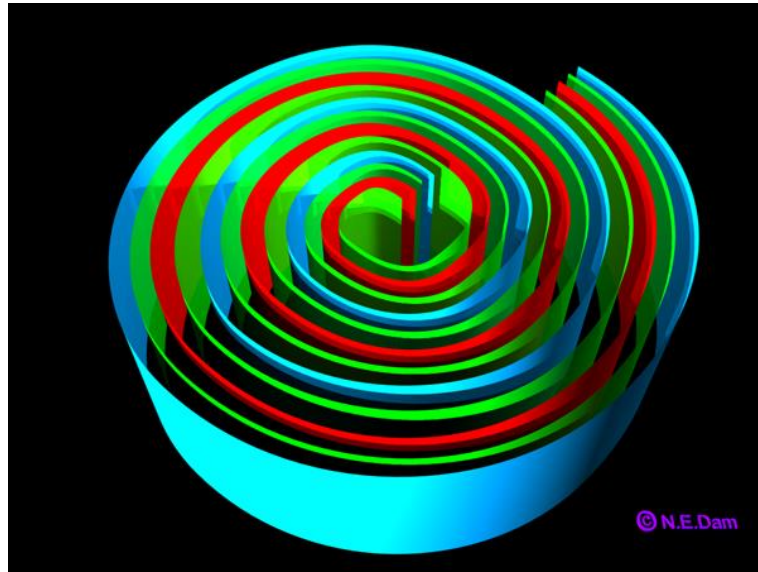


Fig.6.5.1 (N.E. Dam ©). A rolled-up capacitor. Plates a and b are shown red and blue. The green layers are made of an insulating material.

Fig.6.5.1 shows a rolled-up capacitor. Thereby, it is possible to obtain a large area in a small volume. Please note, that we use both sides of the capacitor plates meaning that the effective area is doubled.

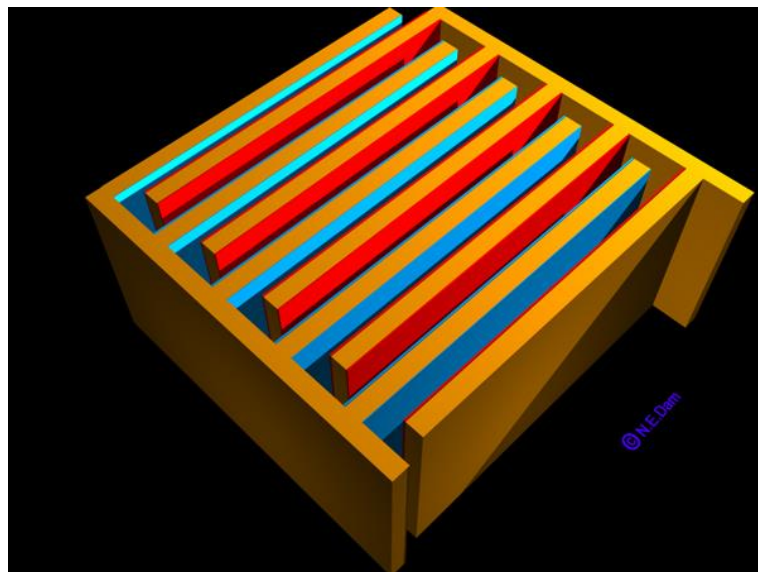


Fig.6.5.2, ref.1 (N.E. Dam ©). A multilayer capacitor.

Fig.6.5.2 shows a multilayer plate capacitor. Here we also use both sides of the capacitor plates meaning that the effective area is doubled. For the outer plates, we only use the one side of the plate. However if the number of plates is large, it is a valid approximation to say that the effective area is doubled.

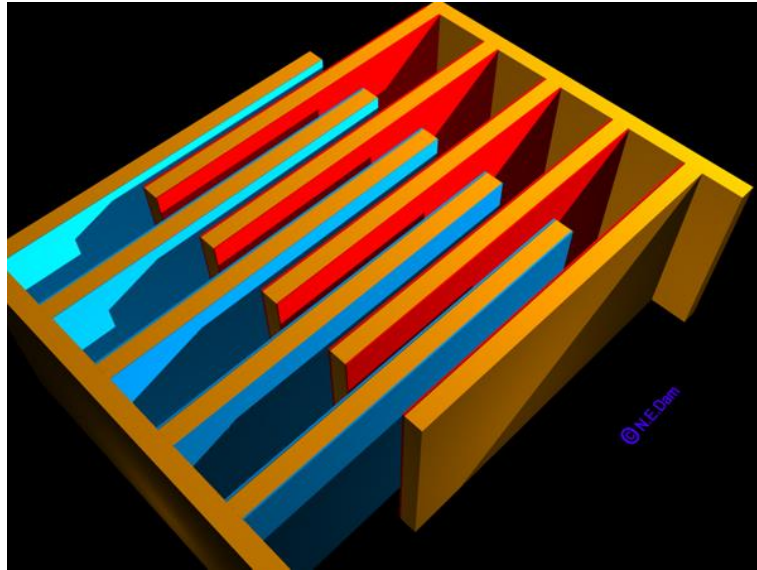


*Fig.6.5.3, ref.1 (N.E. Dam ©).
A multilayer capacitor where the effective area is reduced.*

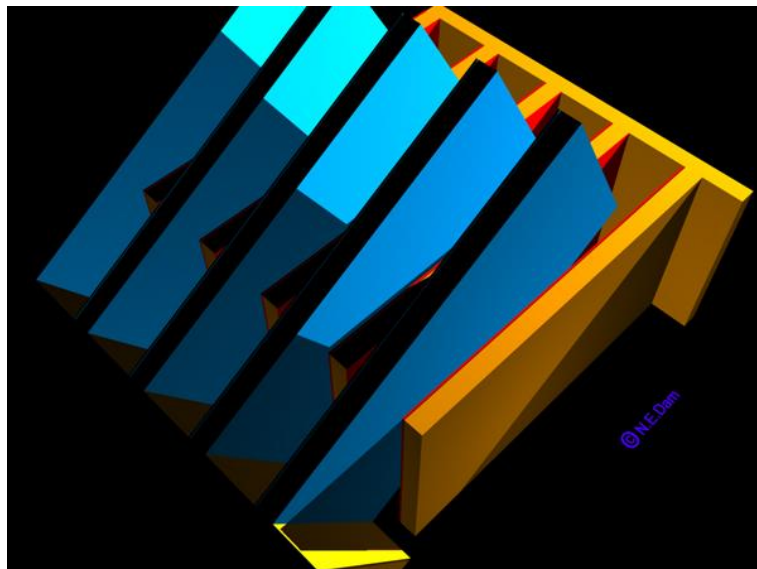


*Fig.6.5.4, ref.1 (N.E. Dam ©). A multilayer capacitor where
the effective area is reduced by rotating one of the capacitor plates.*

Fig.6.5.3 shows that the effective area may be reduced by withdrawing out one of the plates. This may also be realized by rotating one of the plates as shown in [fig.6.5.4](#).

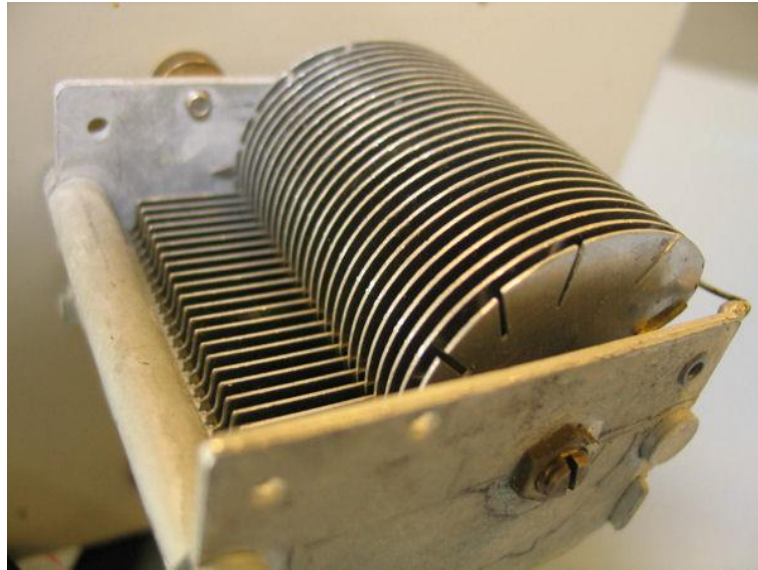


*Fig.6.5.5, ref.1 (N.E. Dam ©).
A practical example of a capacitor with rotatable plates.*

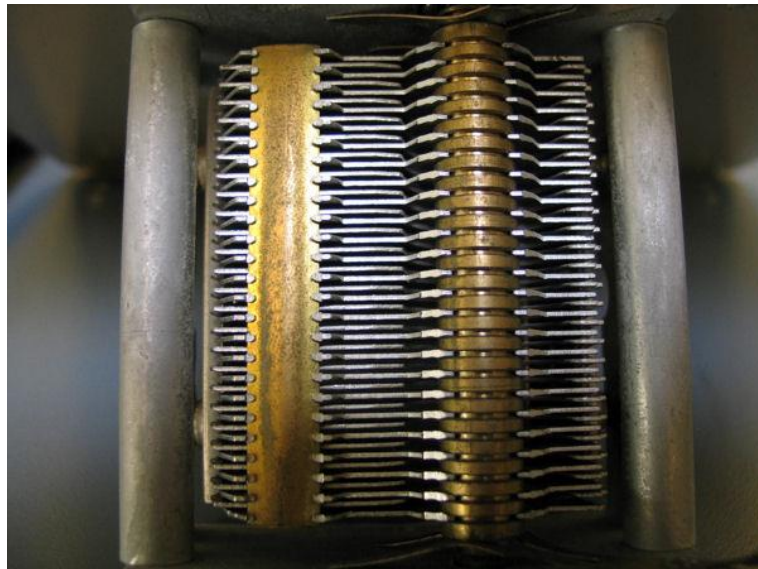


*Fig.6.5.6, ref.1 (N.E. Dam ©). Same capacitor as above. The plates
have been rotated to maximum capacity (maximum effective area).*

Fig.6.5.5 and fig.6.5.6 show a practical example of such a variable capacitance, which may be used in radio sets, transmitters, etc.

6.6 Parallel connection

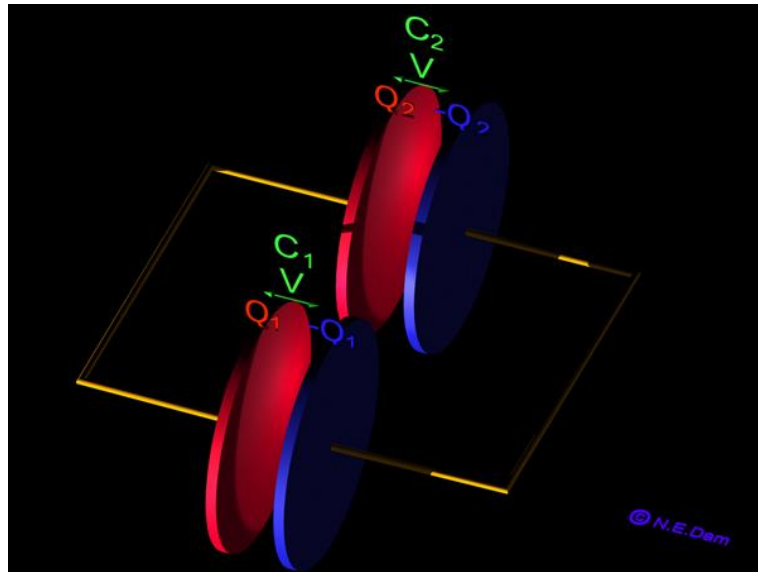


Fig.6.6.1, ref.1 (N.E. Dam ©). A parallel connection of two capacitors.

We connect two different capacitors, C_1 and C_2 , in a parallel connection as shown in fig.6.6.1. In general there will be different charges, Q_1 and Q_2 , on the two capacitors as shown in fig.6.6.1. However, there will be the same potential difference across the two capacitors. If the two capacitors were replaced by just one capacitor, it would have the total charge

$$Q = Q_1 + Q_2 \quad \text{eq.6.6.1}$$

The definition of capacitance gives

$$C = \frac{Q}{V_{ab}} \Rightarrow Q = C V_{ab} \quad \text{eq. 6.6.2}$$

If we use eq.6.6.2 on both sides of eq.6.6.1, we find

$$CV = C_1V + C_2V \quad \text{eq.6.6.3} \Rightarrow$$

$$C = C_1 + C_2 \quad \text{eq.6.6.4}$$

This may be generalized to more capacitors.

6.7 Series connection

We connect two different capacitors, C_1 and C_2 , in a series connection as shown in fig.6.7.1. In general, there will be different voltage drops, V_1 and V_2 , across the two capacitors as shown in fig.6.7.1.

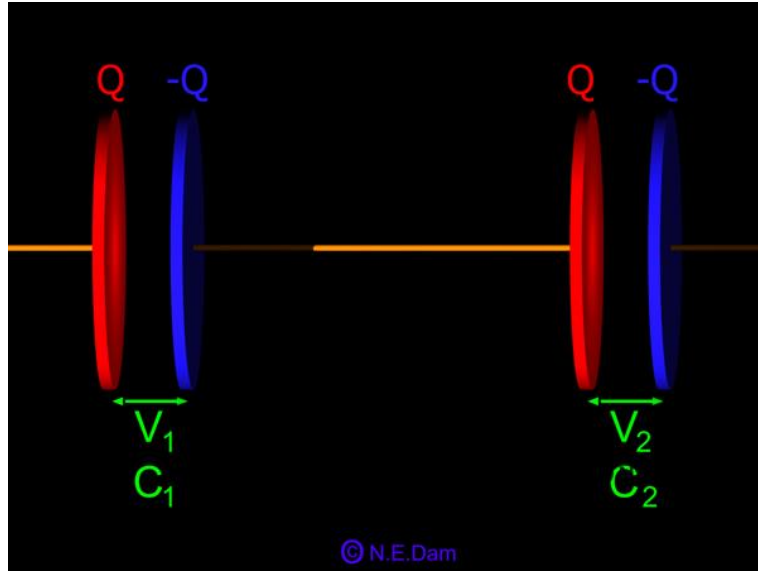


Fig.6.7.1, ref.1 (N.E. Dam ©). A series connection of two capacitors.

However, there will be the same amount of charge on the two capacitors¹. If the two capacitors were replaced by just one capacitor, it would have the charge Q and voltage drop

$$V = V_1 + V_2 \quad \text{eq.6.7.1}$$

The definition of capacitance gives

$$C = \frac{Q}{V_{ab}} \Rightarrow V_{ab} = \frac{Q}{C} \quad \text{eq. 6.7.2}$$

If we use eq.6.7.2 on both sides of eq.6.7.1, we find

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} \quad \text{eq. 6.7.3} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{eq. 6.7.4}$$

This may be generalized to more capacitors.

¹ The reason why we have the same amount of charge on the two capacitors is as follows:

We connect a battery across two capacitors with no charge to begin with. The negative pole of the battery places n electrons on the negative plate of C_2 giving the charge $-Q$. If we place a Gaussian surface around the whole capacitor C_2 , we find that the total charge inside C_2 must be zero because we have no flux out of C_2 when the two capacitor plates are close together. Hence we have $+Q$ on the positive plate of C_2 . When this charge is created, n electrons must leave C_2 and travel to the negative plate of C_1 creating the charge $-Q$ here and $+Q$ on the positive plate of C_1 .

6.8 Dielectric materials

Polarization of a dielectric material

The insulator placed in between the two capacitor plates is called a dielectric material. If we place a dielectric material in front of a negatively charged insulating rod, the molecules in the dielectric material will be polarized as shown in fig.6.8.1.

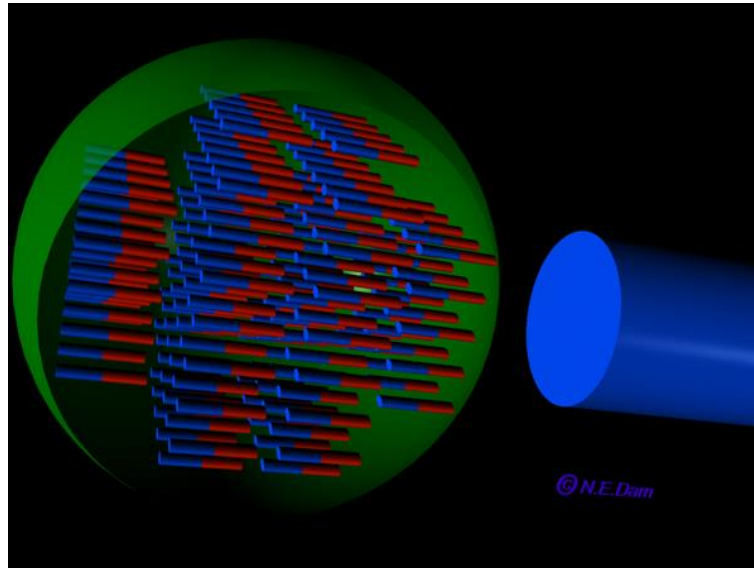


Fig.6.8.1, ref.1 (N.E. Dam ©). Polarization of a dielectric sphere.

The blue rod to the right in the figure has a negative charge. It generates an E-field. This E-field creates dipoles (shown as small blue/red rods) inside the (green) dielectric material. Dipoles are introduced basically in chapter 2. In chapter 4 we derived the electric potential energy for a dipole in an external E-field (an E-field not produced by the dipole itself). We found that the dipole obtains minimum electric potential energy when the dipole is co-parallel with the external E-field. The dipoles in the dielectric material will therefore try to rotate and try to be co-parallel with the external E-field (the E-field from the blue charged rod on the right side of the figure). We see that just below the surface of the insulating sphere we have a negatively charged layer facing away from the blue rod. On the other side of the insulating sphere we have a positively charged layer facing towards the blue rod.

This is analogous to the electric induction described in chapter 2 and shown in fig.6.8.2 for a metal sphere in front of a negative insulating rod. However, in fig.6.8.2 the induced charge on the surface of the metal sphere is due to the movement of “free” conduction electrons in a metal.

In fig.6.8.1 the induced charge is due to the displacement of “bound” electrons within the molecules. The electrons behave – in a simplified way – as if they were bound by an elastic restoring force. The external E-field causes an extra electric force $\vec{F}_E$ acting on the electrons and pulling the electrons in direction of $\vec{F}_E$ almost as if a spring is pulled away from its equilibrium position.

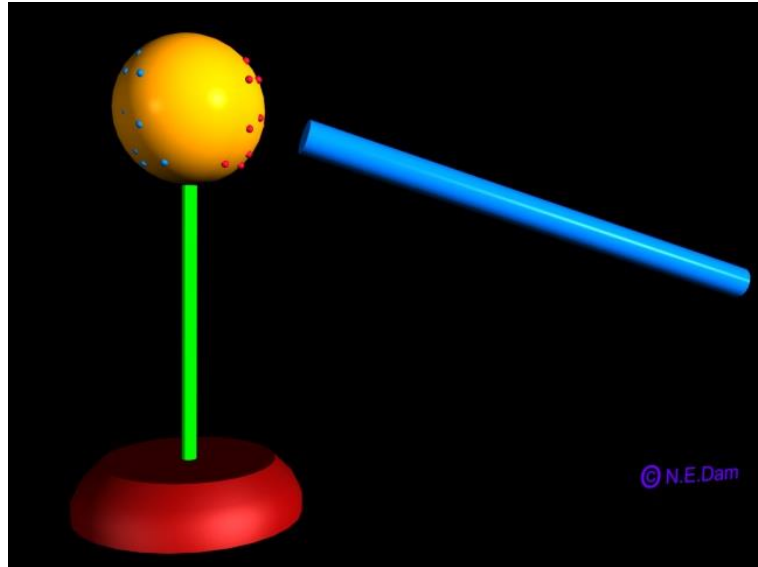


Fig.6.8.2, ref.1 (N.E. Dam ©). Charging by induction. The metal sphere is charged by induction from the rod as described in chapter 2.

Inserting a dielectric material in between the capacitor plates

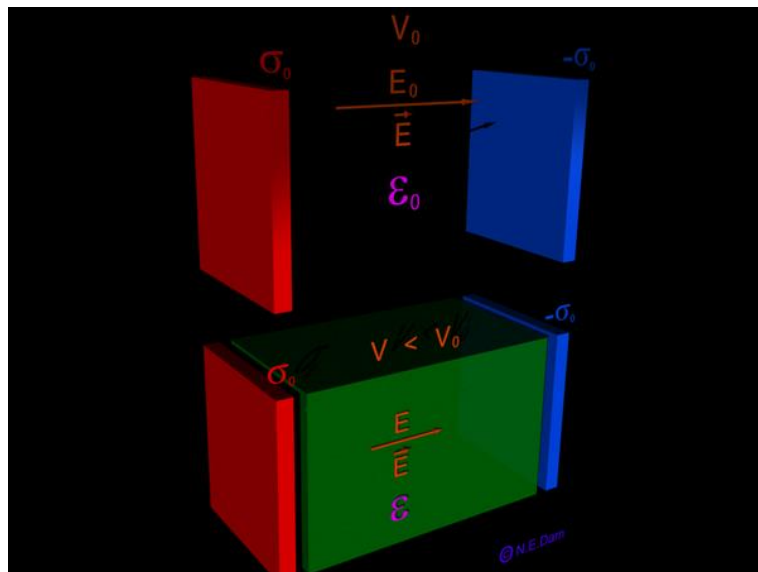


Fig.6.8.3, ref.1 (N.E. Dam ©).
Placing a dielectric material in between the capacitor plates.

Fig.6.8.3 shows what happens, when we place a dielectric material in between the capacitor plates. In the upper part of the figure, we have a vacuum between the capacitor plates. The voltage across the two plates is V_0 and the E-field is from the positive plate to the negative plate. According to the discussion previous to eq.6.2.1, the magnitude of the E-field is

$$E_0 = \frac{\sigma_0}{\epsilon_0} \quad \text{eq. 6.8.1}$$

where σ_0 is the surface charge density on the positive plate. The capacitor is now disconnected from any circuits. Hence Q must be constant.

As shown in the lower part of the figure, we now insert a dielectric material. Now the E-field magnitude changes to

$$E = \frac{\sigma_0}{\epsilon} \quad \text{eq. 6.8.2}$$

Since $\epsilon > \epsilon_0$, we have $E < E_0$. This causes the potential difference V_{ab} to decrease from V_0 to $V < V_0$. A constant Q and a decreasing V_{ab} *implies*, that the capacitance must increase according to

$$C \equiv \frac{Q}{V_{ab}} \quad \text{eq. 6.2.5}$$

The dielectric constant K is defined as the ratio between C and C_0

$$K = \frac{C}{C_0} = \frac{V_0}{V} = \frac{E_0}{E} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \quad \text{eq. 6.8.3}$$

$K = \epsilon_r$ is also called the relative permittivity. It is typically in the order of 2-3 for common insulators.

We have the following permittivities to remember :

- ϵ is the absolute permittivity for a dielectric material
- ϵ_0 is the permittivity for vacuum
- ϵ_r is the permittivity relative to vacuum (the relative permittivity)

The dielectric material - the insulator - has the following advantages :

- It prevents short circuits
- It increases the mechanical stability
- It increases the capacitance
- It increases the breakthrough voltage

Fig.6.8.4 shows what happens with the dielectric material, when it is inserted in between the two capacitor plates.

The electrons cannot move through the material since it is an insulator.

However, the electrons may be displaced within each molecule creating a positive and a negative end of each molecule. If it is possible, the molecules will rotate in such a way that the dipoles associated with the molecules will try to be parallel with the E-field in between the capacitor plates. Inside the dielectric material, we see that the positive and negative ends of the dipoles are facing each other. Hence the net charge will be zero inside the dielectric material.

However, we see that we have a lot of uncompensated negative charge just below the surface of the dielectric material facing towards the positive capacitor plate. We also have a lot of uncompensated positive charge just below the surface of the dielectric material facing towards the negative capacitor plate.

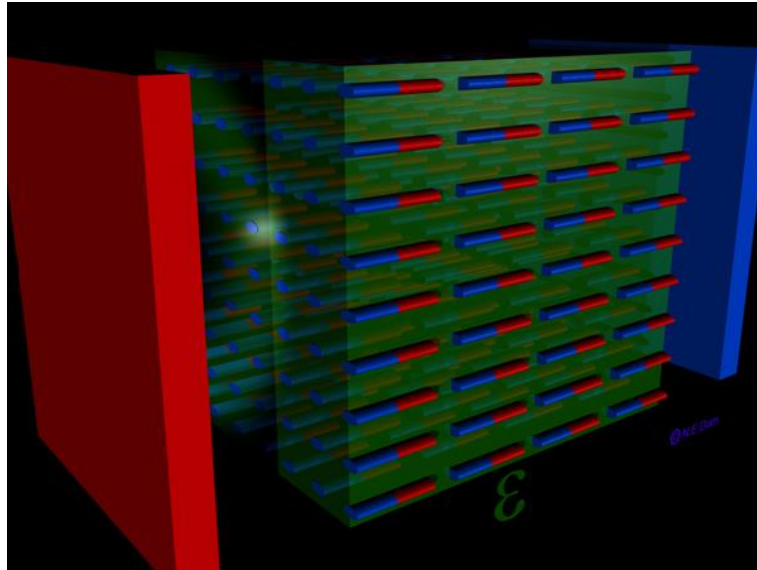


Fig.6.8.4, ref.1 (N.E. Dam ©). Polarization of the dielectric material in-between the two capacitor plates.

The effect of inserting the dielectric material in between the two capacitor plates is the following:

1. It creates a negative layer of charge density $-\sigma_p$ just below the surface of the dielectric material facing towards the positive capacitor plate. The layer of charge is called *polarization charge*, since it is a result of the polarization of the molecules.
2. It creates a positive layer of charge density σ_p just below the surface of the dielectric material facing towards the negative capacitor plate.
3. Inside the dielectric material we have no net charge.

The above (1-2) explains the name “dielectric material”.

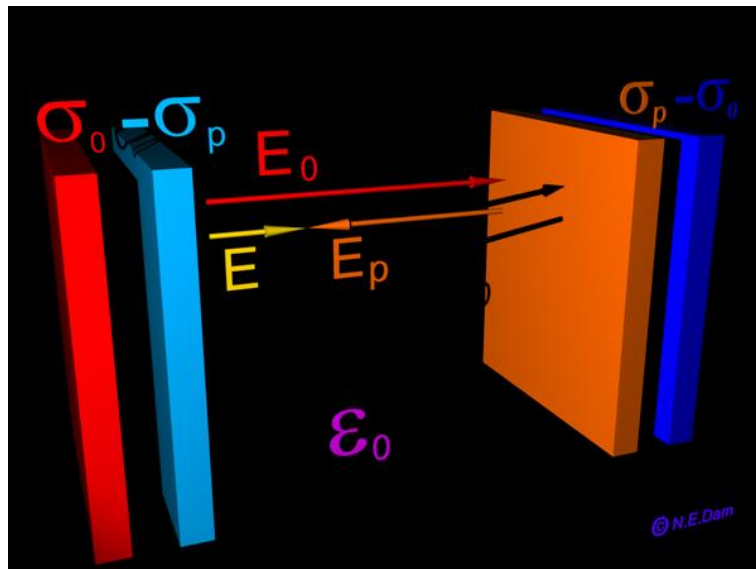


Fig.6.8.5, ref.1 (N.E. Dam ©). We use superposition of E-fields and find that the total E-field is the sum of the E-field from the capacitor plates and the E-field from the polarization double layer.

Instead of inserting the dielectric material of permittivity ϵ we could remove the dielectric material and insert two thin layers of charge density $-\sigma_p$ and σ_p in vacuum as shown in fig.6.8.5. We now have two equivalent capacitor models:

1. *The physical model.* We insert a dielectric material of permittivity ϵ as shown in fig.6.8.3. The E-field magnitude will be $E = \frac{\sigma_0}{\epsilon}$ eq. 6.8.2
2. *The mathematical model.* We remove the dielectric material. Then we have a vacuum. We now insert two thin layers of charge density $-\sigma_p$ and σ_p in the vacuum as shown in fig.6.8.5. We will see below that the E-field magnitude is $E = \frac{\sigma_0 - \sigma_p}{\epsilon_0}$ eq. 6.8.5

Calculation of the total E-field in between the capacitor plates

The total E-field may be calculated using the mathematical model shown in fig.6.8.5.

The total E-field in between the capacitor plates is the sum of two E-fields:

1. The field $\vec{E}_0$ which is due to the two capacitor plates. This E-field is directed to the right. The magnitude is $\frac{\sigma_0}{\epsilon_0}$
2. The field $\vec{E}_p$ which is due to the double layer of polarization charge. This E-field is directed to the left. The magnitude is $\frac{\sigma_p}{\epsilon_0}$

The total E-field is then the sum of these two E-fields:

$$\vec{E} = \vec{E}_0 + \vec{E}_p \quad \text{eq.6.8.4}$$

Since $\vec{E}_p$ is directed opposite to $\vec{E}_0$ we now see why the resulting E-field has a smaller magnitude than $\vec{E}_0$.

$$|\vec{E}| = |\vec{E}_0| - |\vec{E}_p| = \frac{\sigma_0 - \sigma_p}{\epsilon_0} \quad \text{eq. 6.8.5}$$

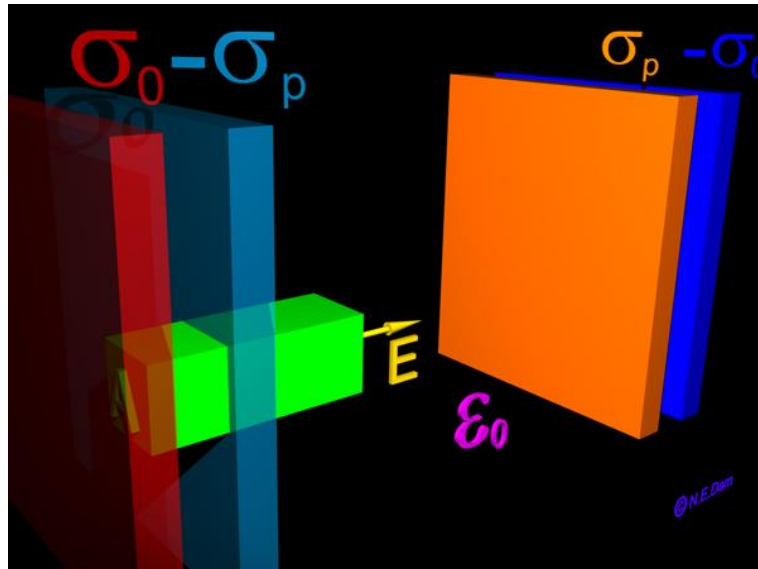


Fig.6.8.6, ref.1 (N.E. Dam ©).
Calculation of the E-field using Gauss's law.

This E-field magnitude may also be calculated using superposition of E-fields and Gauss's law as shown in fig.6.8.6. We insert a Gaussian surface (the green box where the end surface area is A). The flux out of the box is simply EA. The charge inside the

box is the sum of the charge from the capacitor plate and the charge from the polarization layer.

$$EA = \frac{(\sigma_0 - \sigma_p)A}{\epsilon_0} \Rightarrow E = \frac{(\sigma_0 - \sigma_p)}{\epsilon_0} \text{ eq. 6.8.6}$$

which is identical with eq.6.8.5.

Calculation of the polarization charge density

The E-field magnitude found in the physical model is $\frac{\sigma_0}{\epsilon}$.

The E-field magnitude found in the mathematical model is $\frac{\sigma_0 - \sigma_p}{\epsilon_0}$

These two E-fields must be identical:

$$\begin{aligned} \frac{(\sigma_0 - \sigma_p)}{\epsilon_0} &= \frac{\sigma_0}{\epsilon} \text{ eq. 6.8.7} \Rightarrow \\ \epsilon(\sigma_0 - \sigma_p) &= \epsilon_0 \sigma_0 \Rightarrow \sigma_0(\epsilon - \epsilon_0) = \epsilon \sigma_p \Rightarrow \sigma_0 \left(1 - \frac{\epsilon_0}{\epsilon}\right) = \sigma_p \Rightarrow \\ \sigma_p &= \sigma_0 \left(1 - \frac{1}{\epsilon_r}\right) \text{ eq. 6.8.8} \end{aligned}$$

The polarization charge $\sigma_p A$ is also called the bound charge because it is bound on the surface of the insulator.

The charge $\sigma_0 A$ on the conductor surface is called the free charge because it is free to move.

6.9 Energy storage

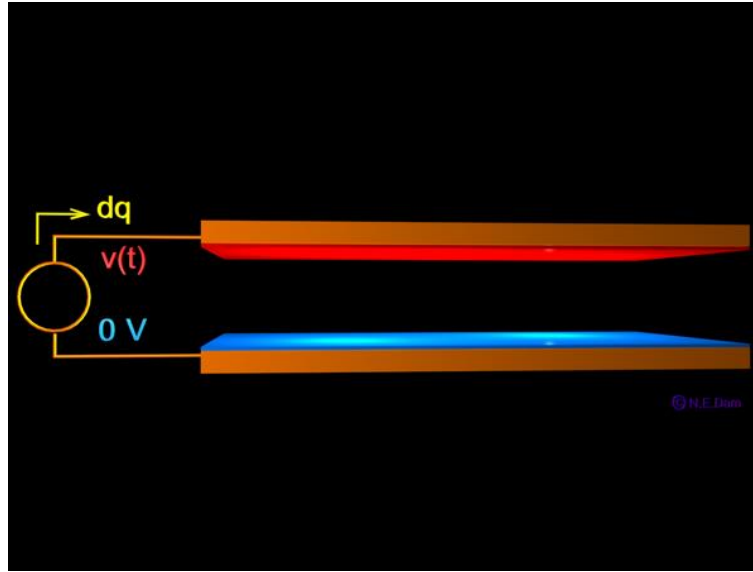


Fig.6.9.1, ref.1 (N.E. Dam ©). Charging a capacitor.

Charging a capacitor will increase the voltage from 0 to the final value V after some time. At time t , the voltage is $v(t) = \frac{q(t)}{C}$. While charging, the source has to do some work to separate the $+$ and $-$ charges and place the charges on the capacitor plates. We define the $-$ plate to have zero potential as shown in fig.6.9.1. Then the $+$ plate has the potential $v(t)$.

Short repetition of the relevant part of chapter 4

According to chapter 4, it requires the following amount of work if q_0 is moved from infinity to a position in between $q_1, q_2, q_3, \dots$

$$U_{\text{pot}}(q_0) = \frac{1}{4\pi\epsilon_0} \left\{ \frac{q_0 q_1}{r_{01}} + \frac{q_0 q_2}{r_{02}} + \frac{q_0 q_3}{r_{03}} + \dots \right\} = \frac{q_0}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_{0i}}$$

According to chapter 5 we have

$$V = \frac{U_{\text{e pot}}}{q_0}$$

This means that the work required to move q_0 from infinity to a position in between $q_1, q_2, q_3, \dots$ may be written as

$$W = U_{\text{pot}, q_0} = q_0 V \quad \text{eq.6.9.1}$$

where V is the potential at the position of q_0 in between $q_1, q_2, q_3, \dots$

Charging the capacitor

According to the above remarks, it requires the following amount of work to move the small charge dq from the potential zero to the potential $v(t)$

$$dW = U_{\text{pot}, dq} = dq v(t) \quad \text{eq.6.9.2}$$

Therefore, the source has to do the following work to transfer the charge dq from potential zero to the potential $v(t)$

$$dW = v(t) dq = \frac{q(t)}{C} dq \quad \text{eq. 6.9.3}$$

We find

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \frac{Q^2}{2} = \frac{1}{2} C V^2 \quad \text{eq. 6.9.4}$$

This is the same as *the amount of energy stored in the capacitor.*

The energy density in a plate capacitor

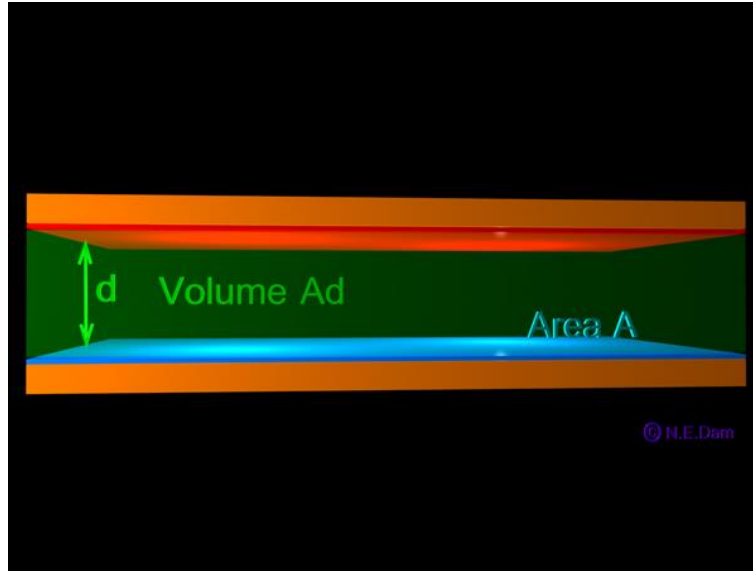


Fig.6.9.2, ref.1 (N.E. Dam ©). The volume inside a capacitor.

The energy density in a plate capacitor in vacuum may be found if we divide the above energy by the volume in between the capacitor plates, see [fig.6.9.2](#).

$$u = \frac{\frac{1}{2} C V^2}{A d} = \frac{\frac{1}{2} \epsilon_0 \frac{A}{d} (Ed)^2}{A d} = \frac{1}{2} \epsilon_0 E^2 \quad \text{eq. 6.9.5}$$

Of course if the vacuum is substituted by an insulator having another value than ϵ_0 for the permittivity we get

$$u = \frac{1}{2} \epsilon E^2 \quad \text{eq. 6.9.6}$$

Please note that the energy density is due to a redistribution of the charges around the considered volume. The source has to do the work $\frac{1}{2} C V^2$ in order to charge the capacitor and create the E-field inside the capacitor. Hence, the E-field represents an amount of energy. This E-field exists in the volume Ad , see [fig.6.9.2](#). Therefore, we consider the density of energy in [eq.6.9.5](#).

Field energy is a useful concept in e.g. electro-magnetic waves, where electric and magnetic fields are considered to have an energy density in vacuum.

Problems

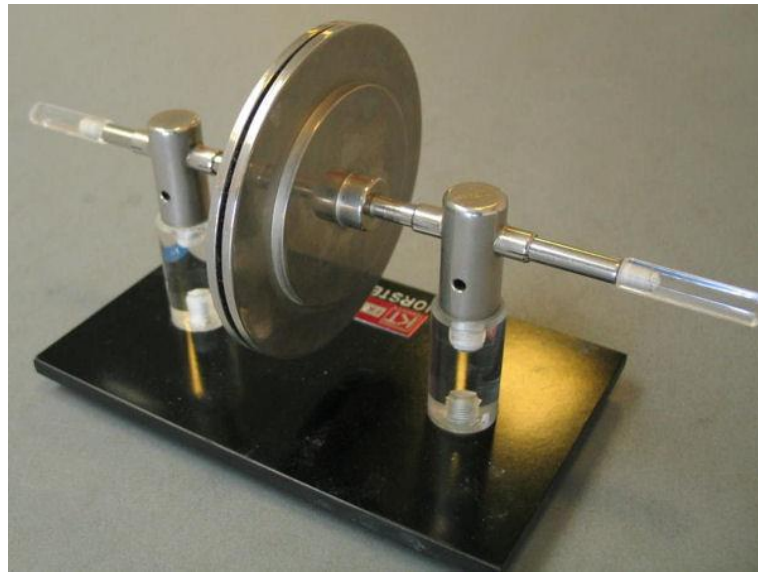


Fig.6.1a ref.1 (N.E. Dam ©). A capacitor for demonstrations.

Pr.6.1 An experimental capacitor

1. Measure the radius R of the capacitor plate (alternatively use $R = 0.06$ m if you have no access to the capacitor). The vacuum permittivity is $\epsilon_0 = 8,854 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$. Now, calculate² C_{c2a} and C_{c4a} for the capacitor in fig.6.1a if $d = 2$ mm and if $d = 4$ mm.
2. Measure C_{m2a} and C_{m4a} without insulator if $d = 2$ mm and if $d = 4$ mm.
3. Compare the calculated and the measured values (C_{c2a} with C_{m2a} and C_{c4a} with C_{m4a}) and try to explain why calculated and the measured values might differ.
4. Measure C_{m2p} with a 2 mm acrylic plate.
5. Measure C_{m2p2a} with a 2 mm acrylic plate plus 2 mm air, i.e. $d = 4$ mm in total.
6. Find the relative permittivity using C_{m2p} and C_{m2a}
7. Find the absolute permittivity.
8. Find the E-field in between the capacitor plates if we connect to a 3V DC power supply. $V_0 = 3$ V.
9. Find Q by using C_0 and V_0
10. Find the charge density σ_0
11. Find the E- field using σ_0 and ϵ_0
12. Find the reduced voltage V when inserting a 2 mm acrylic plate between the capacitor plates.
13. Find the reduced E-field $E = V/d$

² Abbreviations: c = calculated , m = measured, 2a = 2 mm air/vacuum, 2p = 2 mm acrylic plate.

Test

Capacitor without dielectric (without insulator)

1! (10p) Please write the definition of capacitance C

2i (25p) Plate capacitor (permittivity ϵ_0 , charge density σ , distance d , area A). Derive V_{ab} using the E-field and d and then find C .

Capacitor with dielectric

3! (20p) Define the dielectric constant $K=\epsilon_r$ using the capacitance with and without dielectric. Also show how K may be expressed using voltages and permittivities with and without dielectric material.

4 (10p) Write the total E-field in between the capacitor plates using σ_0 and ϵ .

5 (35p) The insulator is replaced by vacuum and two layers of polarization charge with density $-\sigma_p$ and $+\sigma_p$. These polarization layers generate an E-field (E_p) directed opposite to the E-field from the capacitor plates (E_0). Find the resulting E-field.