

7

Dielectric materials

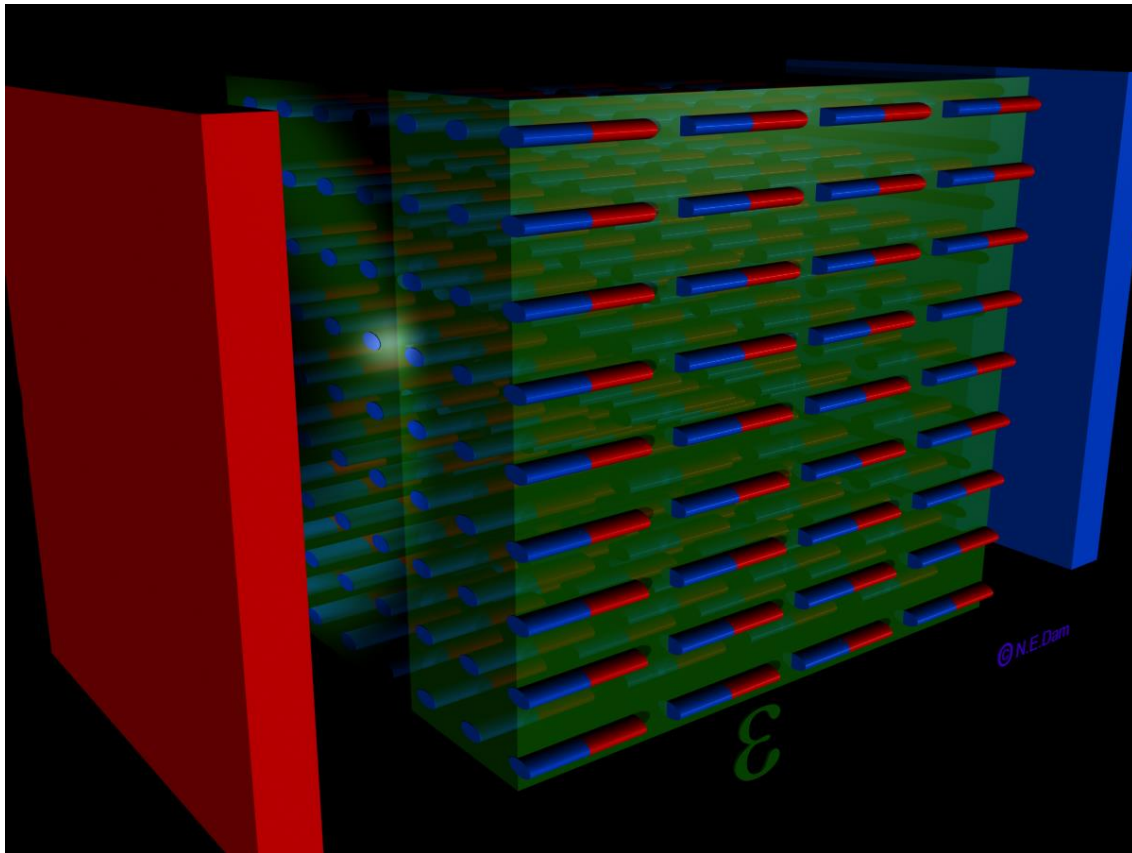


Fig.7.2.1 (N.E. Dam ©). Polarization of the molecules in a dielectric material in between two capacitor plates. The size of the molecules is, of course, exaggerated very much.

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7.1 Introduction

It is recommended to read chapter 2 and 6 before reading this chapter. From chapter 6 we know, that a capacitor consists of two capacitor plates and we also know that the insulator placed in between the two capacitor plates is called a dielectric material. In chapter 6 we have considered the capacitor and learned how to calculate the capacitance of the capacitor with and without the dielectric material in between the capacitor plates. In chapter 7 we will consider the dielectric material in more detail and we will see how the electric E-field inside the dielectric material will change because of the polarization of the material. We will introduce two “new” electric fields, the D-field and the P-field.

- First, in section 7.2, we have an introduction to different dielectric materials.
- In section 7.3, we define the so-called D-field by $\vec{D} \equiv \epsilon_0 \vec{E}_{\text{Ext}}$, where $\vec{E}_{\text{Ext}}$ is the external E-field from the capacitor plates without the dielectric material, the so-called vacuum E-field. We also consider Gauss's law for the D-field.
- Then we continue in section 7.4 with the P-field, the so-called dielectric polarization. It is defined as the electric dipole moment per volume, $\vec{P} \equiv \frac{\overrightarrow{dp_E}}{dV}$. The volume element dV should be large compared to the molecules, but small compared to the entire volume of the material, see fig.7.4.1. It turns out that this P-field is directly related to the polarization E-field, $\vec{P} = -\epsilon_0 \vec{E}_{\text{Pol}}$.
- In section 7.5 we find the total electric E-field, which is a sum of the vacuum E-field and the polarization E-field, $\vec{E} = \vec{E}_{\text{Ext}} + \vec{E}_{\text{Pol}}$.
- In section 7.6 we consider the fields in anisotropic dielectric materials.
- In section 7.7 we find the relation $\vec{E} = \vec{D}/\epsilon$ which is valid in isotropic, linearly dielectric materials.
- In section 7.8 we see how to calculate the E-fields in different layers in a sandwich with more dielectric materials using the common value for the D-field through the different layers.
- Finally, in section 7.9 we see examples of different polarized molecules.

7.2 Polarization of a dielectric material

In this section we will consider an insulating material, a so-called dielectric material, which is placed in between two capacitor plates, so we get an external electric E-field, $\vec{E}_{\text{Ext}}$, across the dielectric material.

When we apply a potential difference between the two capacitor plates, we get an electric E-field across the dielectric material in between the two capacitor plates. This E-field will cause the positive charges in the dielectric material to move in the same direction as the E-field and the negative charge to move in the opposite direction. This is what we call *polarization*. As mentioned, the dielectric material is an insulator. Hence the charges cannot move through the material from the one side to the opposite side of the material. However, the charges can move - to some extent - within each molecule.

Inside a molecule, the positive charges are usually the nuclei, and they are not able to move very much because the atoms are placed in a solid matter¹.

However, some of the electrons in a molecule will often be able to move a greater distance within the same molecule. Therefore, the molecules may develop a negative end, where there is an excess of electrons, and a positive end, where there are a deficit of electrons if we compare with the normal state of the molecules without an external E-field from the capacitor plates. This phenomenon, which is called *polarization* of the molecules is illustrated in fig.7.2.1. This polarization is on a microscopic (molecular) scale.

Let us consider a volume element enclosing several molecules inside the dielectric material. The charge density inside this volume element will remain zero after the polarization of the molecules. For every plus end of a molecule with positive polarization charge, there will be a minus end of the neighbour molecule with negative polarization charge which will compensate for the positive polarization charge. However, as shown in the figure, each of the two end surfaces of the dielectric material, facing the capacitor plates, will develop a layer of polarization charge just below the surface, and thereby the material becomes polarized on a macroscopic scale between the two end surfaces. This will be explained in more detail for different kinds of materials in section 7.9. We consider the following three possibilities:

1. A non-polar molecule (e.g. H_2) becomes polarized because positive charge inside the molecule is displaced in direction of the total E-field vector $\vec{E}$ and negative charge inside the molecule is displaced in the opposite direction.
2. A crystal with positive and negative ions (e.g. NaCl) becomes polarized because the positive ions are displaced in direction of $\vec{E}$ and the negative ions are displaced in the opposite direction.
3. A polarized molecule (e.g. H_2O) is rotated so its electric polarization vector $\vec{p}_E$ becomes coparallel with $\vec{E}$. Thereby, the $\vec{P}$ vector for the polarized material becomes coparallel with $\vec{E}$.

¹ Remarks about solid matter.

The atoms or molecules in a solid matter may be organized in a crystal structure. In that case there is a so-called unit cell. The crystal structure may be build up by repeating the unit cell in the x, y and z directions. The unit cell may be considered as a brick, that can be used for building up the crystal. The unit cell is the smallest unit, that can be used to build up the crystal.

Many materials consist of a large number of relatively small crystals, which are grown together during the solidification process. The small crystals usually have different crystallographic orientation and they are separated by so-called grain boundaries.

If the atoms are not organized in a crystal structure, the material is said to be amorphous. This is the case for many plastic materials and glass.

7.3 The D-field

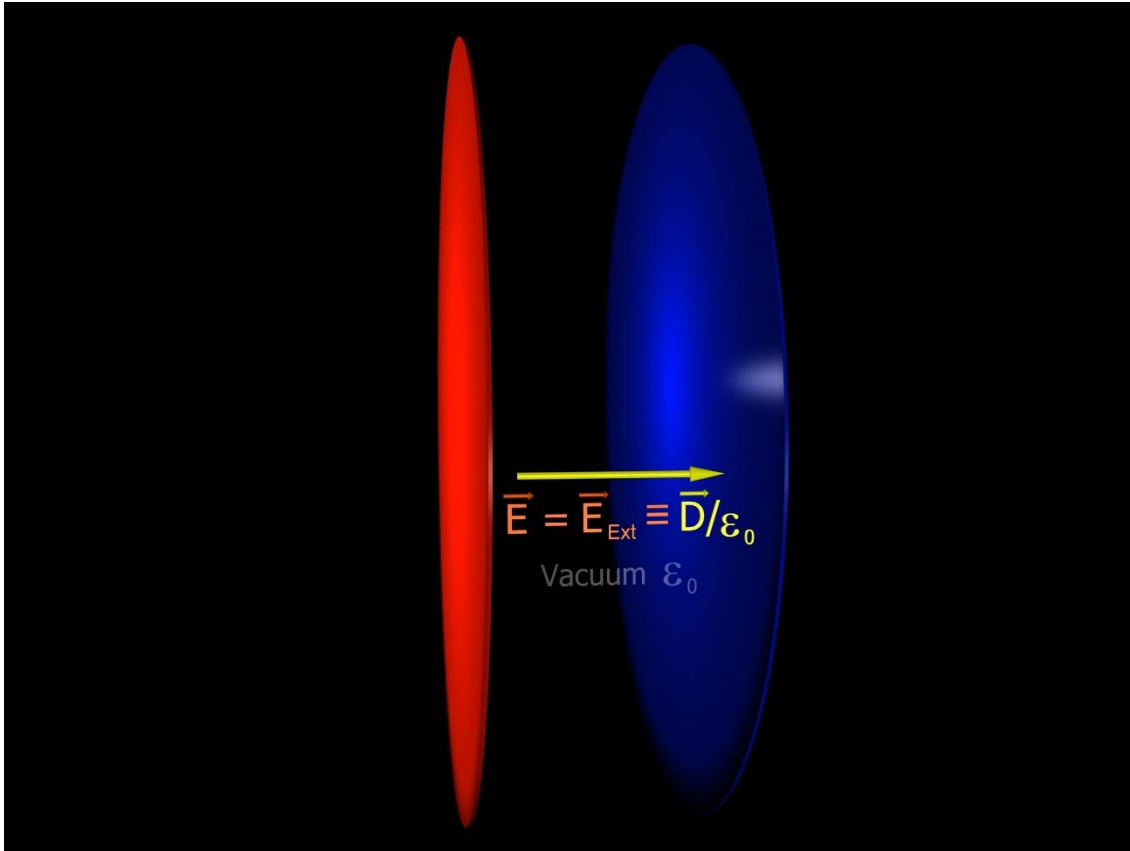


Fig.7.3.1 (N.E. Dam ©). The dielectric material has been replaced by vacuum and we have introduced the D-field.

Fig.7.3.1 shows that the dielectric material in between the two capacitor plates has been replaced by vacuum. The charges on the capacitor plates, Q_{Ext} and $-Q_{Ext}$, creates an external E-field, $\vec{E}_{Ext}$, which is denoted as the vacuum E-field. It is the E-field we get if we remove the dielectric material.

The D-field is now defined as the vacuum E-field times ϵ_0

$$\vec{D} = \epsilon_0 \vec{E}_{Ext} \quad eq. 7.3.1$$

or

$$\vec{E}_{Ext} = \vec{D}/\epsilon_0 \quad eq. 7.3.2$$

The D-field is ϵ_0 times the vacuum E-field that will be there, if we remove the dielectric material or “turn off” all the polarization charges. The D-field is created entirely by the charges on the capacitor plates *outside* the dielectric material, hence the name “External”. The D-field is created by all other charges than the polarization charges in the dielectric material. If we ask “What is the D-field if we have a dielectric material” or “What is the D-field inside the dielectric material”, the answer is:

The D-field inside the dielectric material is ϵ_0 times the E-field we get if we remove the dielectric material or “turn off” all the polarization charges.

There are many synonymous names for the charge on the capacitor plates:

1. *Free charge* indicates, that the charge is free to move through a metal structure.
2. *Real charge* indicates, that the charge is a really physical charge, that can charge a capacitor or flow through a wire.
3. *External charge* (Ext) indicates, that the charge is not a part of the dielectric material and not one of the charges, that can only move inside a molecule.
4. *Normal charge* indicates, that this is the charge we have considered in the electrostatic theory before we introduced the dielectric material in connection with the capacitor.
5. *Vacuum charge* indicates, that this is the charge we have left if we remove all dielectric materials and only have vacuum and metal structures like in chapters 2-5.

All 5 names are - as mentioned - synonymous. In this text we use *External charge (Ext)*.

The E-field lines start and stop on all kinds of charges, external charges and polarization charges.

The D-field lines start and stop only on external charges and ignore the polarization charges.

In the dielectric free world with vacuum and metals considered in the chapters 2-5 we have no polarization charges. We have only what we call the external charges mentioned above. Hence, $\vec{E} = \vec{E}_{\text{Ext}}$ i.e. all E-fields are caused by the external charges. We may therefore substitute $\vec{E}$ by $\vec{D}/\epsilon_0$ everywhere in all equations and formulas in chapters 2-5 because we have no dielectric material, only vacuum.

For example in vacuum we have Gauss's law for the E-field and for the D-field

$$\Phi_E = \oint_{\text{GS}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{Encl}}}{\epsilon_0} = \frac{Q_{\text{Encl Ext}}}{\epsilon_0} \quad \text{eq. 7.3.3}$$

$$\epsilon_0 \Phi_E = \oint_{\text{GS}} \epsilon_0 \vec{E} \cdot d\vec{A} = \Phi_D = \oint_{\text{GS}} \vec{D} \cdot d\vec{A} = Q_{\text{Encl Ext}} \quad \text{eq. 7.3.4}$$

Gauss's law for the D-field is

$$\Phi_D = \oint_{\text{GS}} \vec{D} \cdot d\vec{A} = Q_{\text{Encl Ext}} \quad \text{eq. 7.3.5}$$

Eq.7.3.3 is only valid in vacuum. Air behaves in a very good approximation as a vacuum when it comes to calculating and considering electric fields.

Eq.7.3.5 is valid in vacuum and also in a dielectric material since the D-field ignores the polarization charges from the dielectric material. The D-field is the same inside a dielectric material as if we remove the dielectric material.

The reason why we are interested in the D-field is, that in a dielectric material it may be complicated to calculate the E-field because of the influence of the polarization charges. However, the D-field inside a dielectric material is the same as if we have no dielectric material, hence the D-field is much easier to calculate.

Using Gauss's law from chapter 3, we may find the magnitude of the external E-field in fig.7.3.1

$$E_{\text{Ext}} = \frac{\sigma_{\text{Ext}}}{\epsilon_0} \quad \text{eq. 7.4.6}$$

The above is only valid for the 1D geometry.

We repeat, that the D-field is the ϵ_0 E-field in vacuum meaning that the D-field is the E-field from the external charges. The D-field is not affected by the dielectric material. However, if we insert the dielectric material in between the capacitor plates with the external charges, the D-field *causes* the charges inside the molecules to be displaced as described earlier. This results in the two layers of polarization charge, the so-called double layer of polarization charge, which creates the so-called polarization E-field, $\vec{E}_{\text{Pol}}$.

Once more, the D-field may be calculated as follows:

1. Remove the dielectric material
2. Calculate the vacuum E-field
3. Multiply the vacuum E-field by ϵ_0 to get the D-field $\vec{D} = \epsilon_0 \vec{E}_{\text{Ext}}$

7.4 The P-field

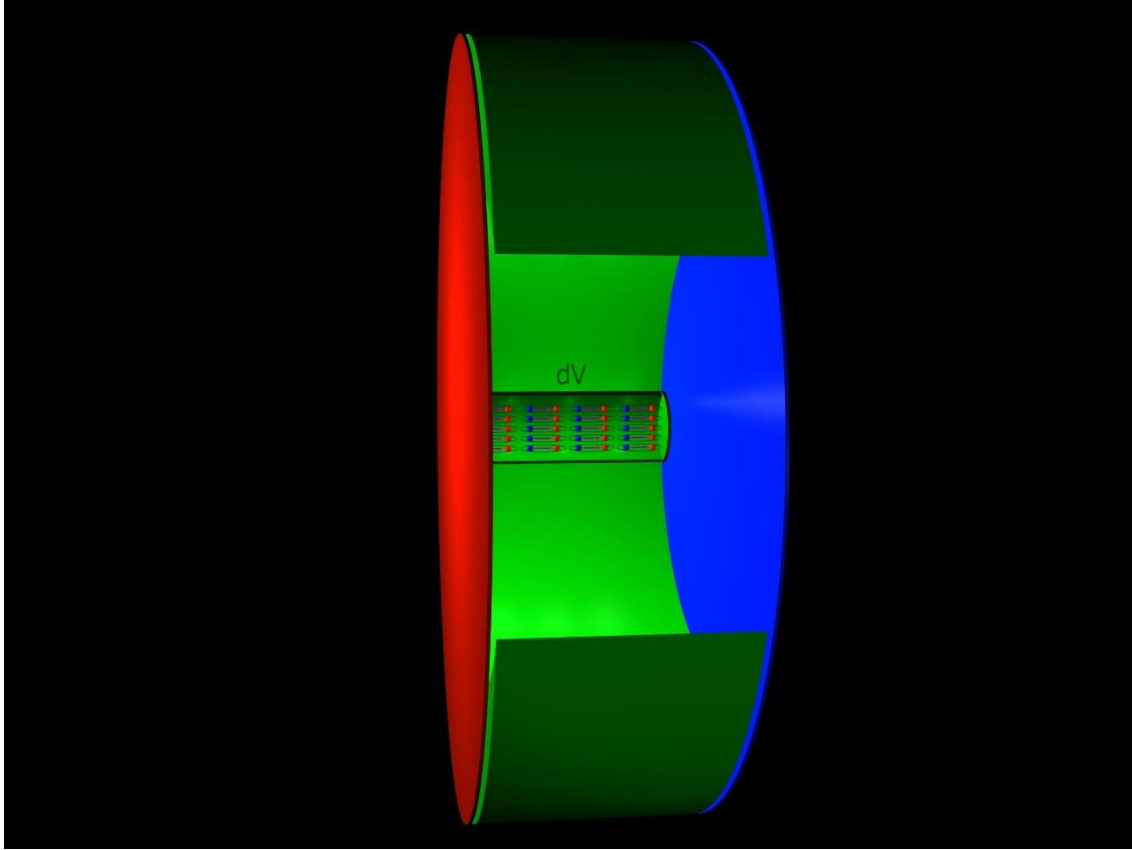


Fig.7.4.1 (N.E. Dam ©). The space between the capacitor plates is completely filled with dielectric material. In the figure, however, some of this material has been removed so that one can "look" into the material and see the volume element dV .

The derivation of the dielectric polarization $\vec{P}$

We consider a volume element dV of the dielectric material, see fig.7.4.1.

In below we will derive the dielectric polarization $\vec{P}$ based on the electric dipole moment for the molecules, see fig.7.4.2. We equivate a polarized molecule by a dipole, that consists of the two charges $-q_{\text{Pol}}$ and q_{Pol} with the internal distance $\vec{d}$. The electric dipole moment for a single molecule is then defined by

$$\vec{p}_{\text{E, molecule}} = q_{\text{Pol}} \vec{d} \quad \text{eq. 7.4.1}$$

The electric dipole moment, $\overline{dp}_{\text{E}}$, for a volume element dV is simply the sum of the electric dipole moments of all the molecules inside dV as shown in fig.7.4.2.

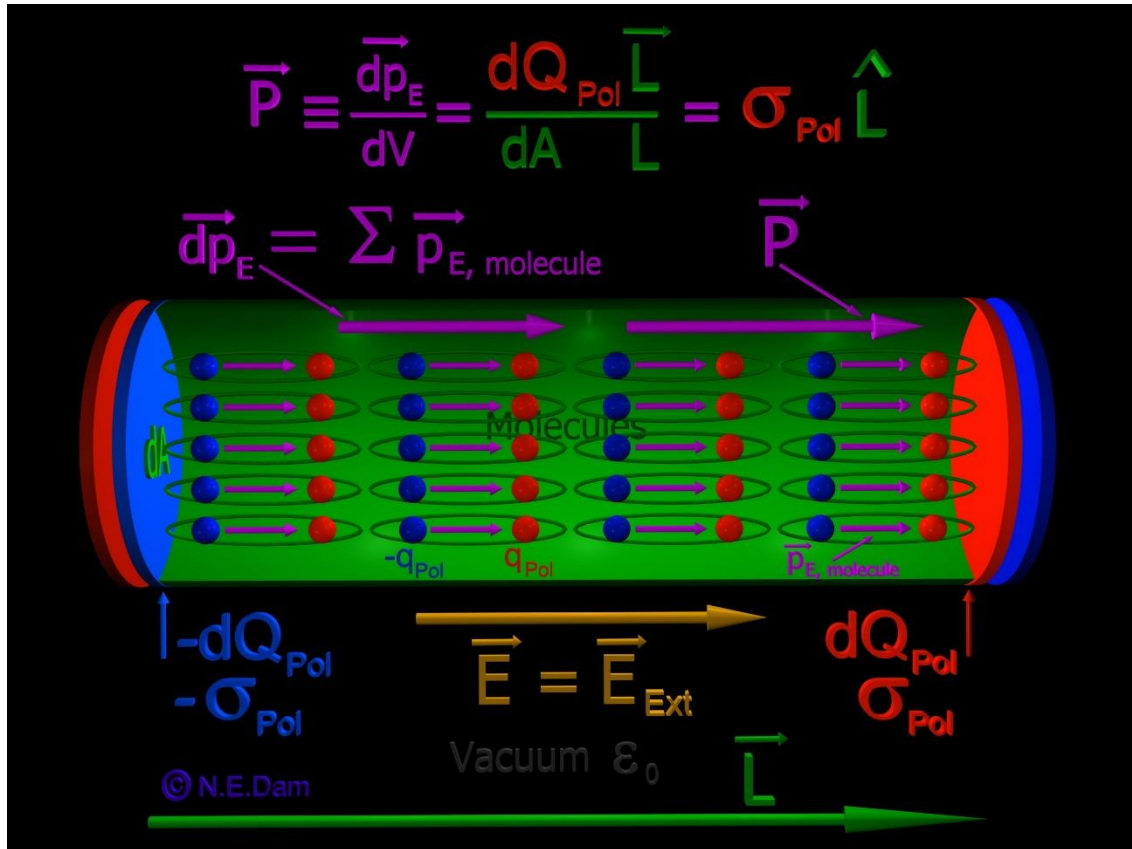


Fig.7.4.2 (N.E. Dam ©). The electric dipole moment $\vec{p}_{E, \text{molecule}}$ for a single molecule, the electric dipole moment $d\vec{p}_E$ for a volume element dV and the definition of the dielectric polarization $\vec{P}$.

$$d\vec{p}_E = \sum_{\text{Molecules within } dV} \vec{p}_{E, \text{molecule}} \quad \text{eq. 7.4.2}$$

The electric dipole moment, $d\vec{p}_E$, for the volume element dV must be proportional to the volume dV . Hence the electric dipole moment per volume must be a constant. We now define:

The dielectric polarization $\vec{P}$ is the electric dipole moment per volume

$$\vec{P} \equiv \frac{d\vec{p}_E}{dV} \quad \text{eq. 7.4.3}$$

The physical meaning of the dielectric polarization

Fig.7.4.2 shows a volume element dV of length L and cross section area dA , it could be a cylinder or a box. The volume element is filled with a dielectric material, which is polarized in direction $\hat{L}$. We place the material in between two charged capacitor plates. As mentioned earlier, the polarization creates two layers of polarization charge $-dQ_{Pol}$ and dQ_{Pol} at the end surfaces of dV . Hence the volume element may be considered as a macroscopic dipole consisting of the two charges $-dQ_{Pol}$ and dQ_{Pol} at internal distance $\vec{L}$.

The word macroscopic means, that we consider something, which is large relative to the molecules. It could for example be $\vec{dp}_E$, which is the total electric dipole moment for all the molecules in the volume element dV . Even if dV is considered as a relative small volume, it will normally consist of a large number of molecules.

The word microscopic means, that we consider something, that is about the same size as a molecule, it could be the electric dipole moment $\vec{p}_{E, \text{molecule}}$ for a single molecule.

The electric dipole moment for the volume element dV is then

$$\vec{dp}_E = dQ_{Pol} \vec{L} \quad \text{eq. 7.4.4}$$

The volume is

$$dV = dA L \quad \text{eq. 7.4.5}$$

where dA and L are the cross-sectional area and the length of dV , respectively.

Thereby we find

$$\vec{P} = \frac{\vec{dp}_E}{dV} = \frac{dQ_{Pol} \vec{L}}{dA L} = \sigma_{Pol} \hat{L} \quad \text{eq. 7.4.6}$$

Hence, the physical meaning of the dielectric polarization is:

The dielectric polarization $\vec{P}$ is a vector

- 1. The direction is the displacement direction of the positive polarization charge*
- 2. The magnitude is the amount of displaced polarization charge per area, where the area is perpendicular to the displacement direction*

The relation between $\vec{P}$ and $\vec{E}_{Pol}$ in 1D geometry

Now we derive the relation between the dielectric polarization $\vec{P}$ and the polarization E-field, $\vec{E}_{Pol}$. The following derivation is valid for a 1D geometry, e.g. a thin plate in between two capacitor plates. The E-field is calculated using the result from chapter 3 "Field inside a double layer of charge", which is a 1D geometry.

The effect of the dielectric material is to create two layers of polarization charge. If we are only interested in the E-field, the dielectric material may be removed and replaced by vacuum and the two layers of polarization charge, $-dQ_{Pol}$ and dQ_{Pol} as shown in fig. 7.4.3. It will give the same E-field as the dielectric material. The E-field created by the two layers of polarization charge is the so-called polarization E-field. Using Gauss's law from chapter 3 we find the magnitude of this E-field

$$E_{Pol} = \frac{\sigma_{Pol}}{\epsilon_0} \quad \text{eq. 7.4.7}$$

From eq. 7.4.6 we have

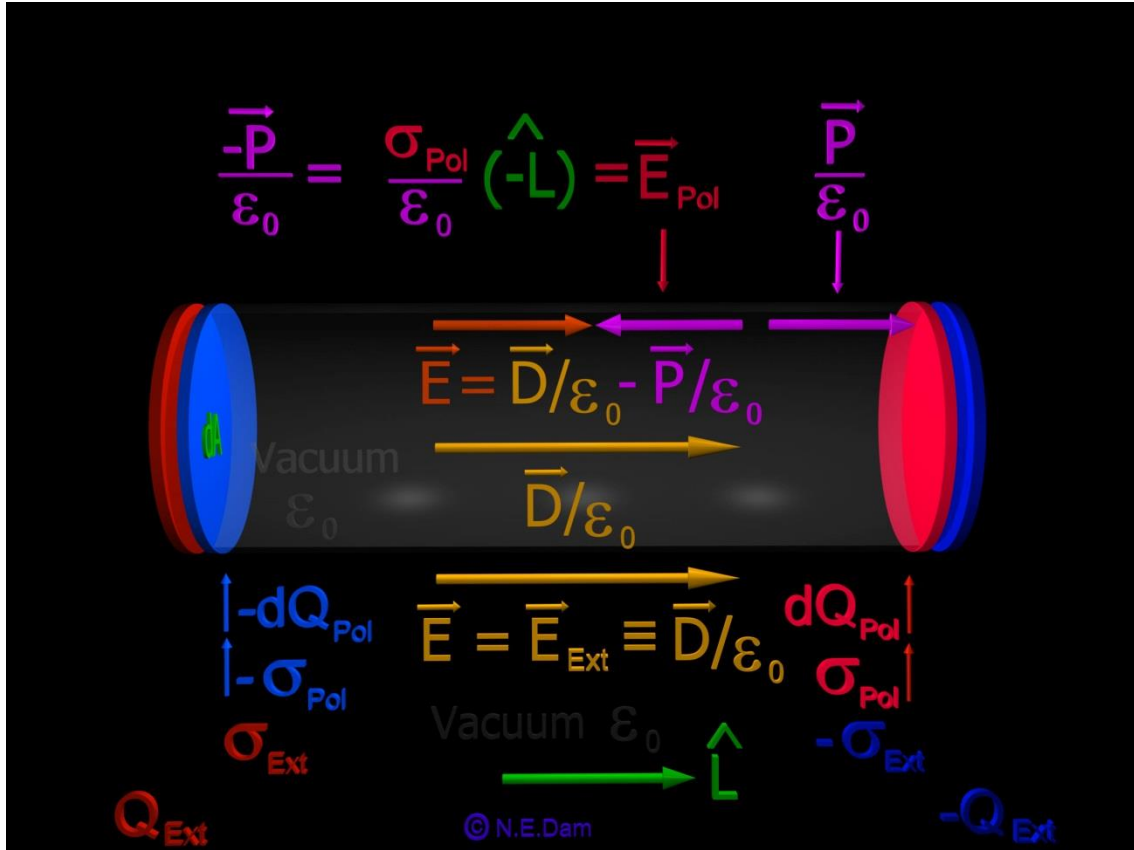


Fig.7.4.3 (N.E. Dam ©). The dielectric material has been replaced by vacuum and two layers of polarization charge, and we have introduced the P-field and the D-field.

$$P = \sigma_{Pol} \quad \text{eq. 7.4.8}$$

Hence we find

$$E_{Pol} = \frac{P}{\epsilon_0} \quad \text{eq. 7.4.9}$$

Fig.7.4.2 shows that $\vec{E}_{Pol}$ is opposite directed relative to $\vec{P}$. We hereby find

The relation between $\vec{P}$ and $\vec{E}_{Pol}$ in a thin plate

$$\vec{E}_{Pol} = -\frac{\vec{P}}{\epsilon_0} \quad \text{eq. 7.4.10}$$

As mentioned above this result is valid for a 1D geometry such as a thin plate of dielectric material in between two capacitor plates. If the dielectric material has another shape, e.g. a sphere, a similar derivation ends up with

The relation between $\vec{P}$ and $\vec{E}_{Pol}$ in a sphere

$$\vec{E}_{Pol} = -\frac{\vec{P}}{3\epsilon_0} \quad \text{eq. 7.4.11}$$

7.5 The total E-field

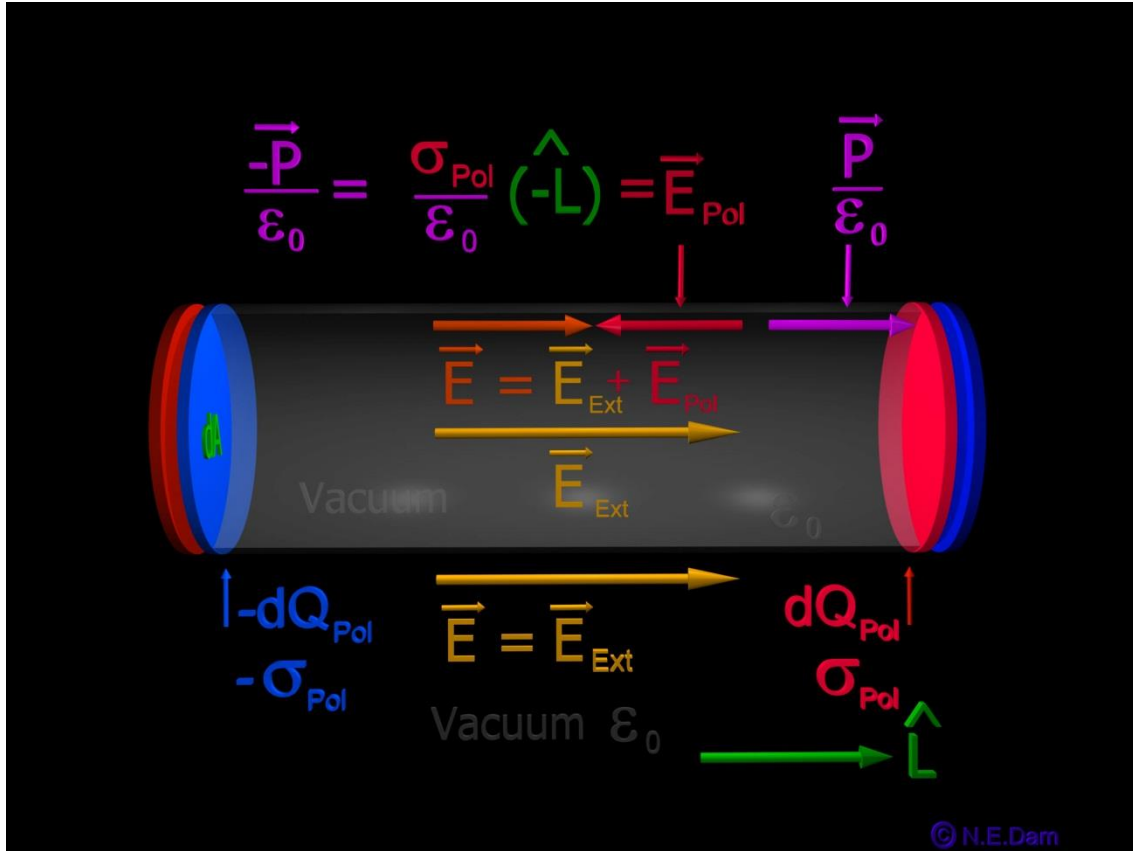


Fig.7.5.1 (N.E. Dam ©). The dielectric material has been replaced by vacuum and two layers of polarization charge.

If we combine

$$\vec{E}_{\text{Pol}} = -\frac{\vec{P}}{\epsilon_0} \quad \text{eq. 7.4.10}$$

and

$$\vec{E}_{\text{Ext}} = \vec{D}/\epsilon_0 \quad \text{eq. 7.3.2}$$

we find for the total E-field as shown in fig.7.4.3

$$\vec{E} = \vec{E}_{\text{Ext}} + \vec{E}_{\text{Pol}} = \frac{\vec{D}}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} \quad \text{eq. 7.5.1}$$

For the magnitudes we have already found

$$E_{\text{Pol}} = \frac{\sigma_{\text{Pol}}}{\epsilon_0} \quad \text{eq. 7.4.7}$$

and

$$E_{\text{Ext}} = \frac{\sigma_{\text{Ext}}}{\epsilon_0} \quad \text{eq. 7.4.6}$$

If we consider fig.7.5.1 we see, that $\vec{E}_{\text{Pol}}$ is directed opposite to $\vec{E}_{\text{Ext}}$. Hence, if we are using the magnitudes of the vectors we find for the total E-field

$$|\vec{E}| = |\vec{E}_{\text{Ext}}| - |\vec{E}_{\text{Pol}}| = \frac{\sigma_{\text{Ext}}}{\epsilon_0} - \frac{\sigma_{\text{Pol}}}{\epsilon_0} \quad \text{eq. 7.5.2}$$

7.6 Anisotropic dielectric materials

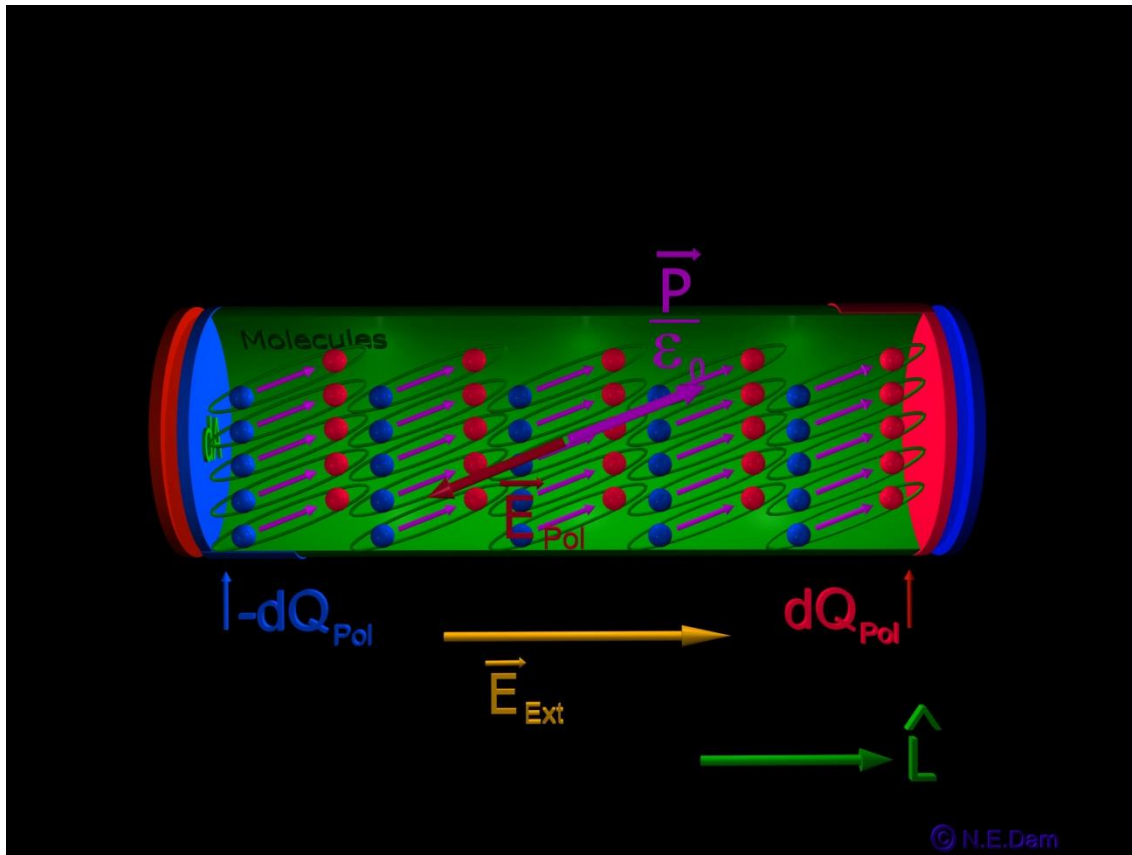


Fig.7.6.1 (N.E. Dam ©). For an anisotropic dielectric material, the polarization charge is generally not displaced in the same direction as the total E-field. The P-field will then be in another direction than the total E-field. This causes, that there will be polarization charge not only at the end surfaces but also at other surfaces.

In all the sections 7.1 to 7.5 we have considered dielectric materials, where the polarization charge was displaced in the same direction as the external E-field and thereby also in the same direction as the total E-field $\vec{E}$. In other words, the $\vec{P}$ -vector was parallel with the $\vec{E}$ vector. Such materials are denoted isotropic dielectric materials.

For isotropic dielectric materials the $\vec{P}$ -vector is parallel with $\vec{E}$

However, in many cases the material is polarized in another direction than total E-field $\vec{E}$. Such materials are denoted anisotropic dielectric materials.

For anisotropic dielectric materials the $\vec{P}$ -vector is in general not parallel with the $\vec{E}$ vector

Fig.7.6.1 shows an anisotropic dielectric material, where the polarization charge is displaced in another direction than the total E-field. The reason for that may be, that the structure in the material forces the polarization charges to move in certain directions, so-called easy directions. E.g., if we consider the structure of NaCl *in fig.7.2.6 we understand, that the ions cannot move in all directions but only in

directions dictated by the crystal structure. It means, that the charges can move only in certain directions. There might be a number of allowed directions for the polarization charge to move. The directions depend on the crystallographic structure. When the external E-field is turned on, the polarization charge “should make a choice” between a number of possible directions. Then the polarization charge will “choose” to move in the direction, which is as close as possible to the total E-field. The $\vec{P}$ -vector will always point in the direction, where the (positive) polarization charge is moving. When the polarization charge moves in another direction than the total E-field, the $\vec{P}$ -vector will be in another direction than $\vec{E}$, see fig.7.6.2. The total E-field may be calculated by

$$\vec{E} = \vec{E}_{\text{Ext}} + \vec{E}_{\text{Pol}} = \frac{\vec{D}}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} \quad \text{eq. 7.5.1}$$

which is the same equation as for the isotropic materials. However, the vectors are not parallel, see fig.7.6.2. In principle we may use

$$E_{\text{Pol}} = \frac{\sigma_{\text{Pol}}}{\epsilon_0} \quad \text{eq. 7.4.7}$$

and

$$E_{\text{Ext}} = \frac{\sigma_{\text{Ext}}}{\epsilon_0} \quad \text{eq. 7.4.6}$$

to calculate the E-field magnitudes as we did for the isotropic materials.

However, since the vectors $\vec{E}_{\text{Ext}}$ and $\vec{E}_{\text{Pol}}$ are not parallel in an anisotropic material, we cannot use eq.7.5.2 to find the total E-field magnitude.

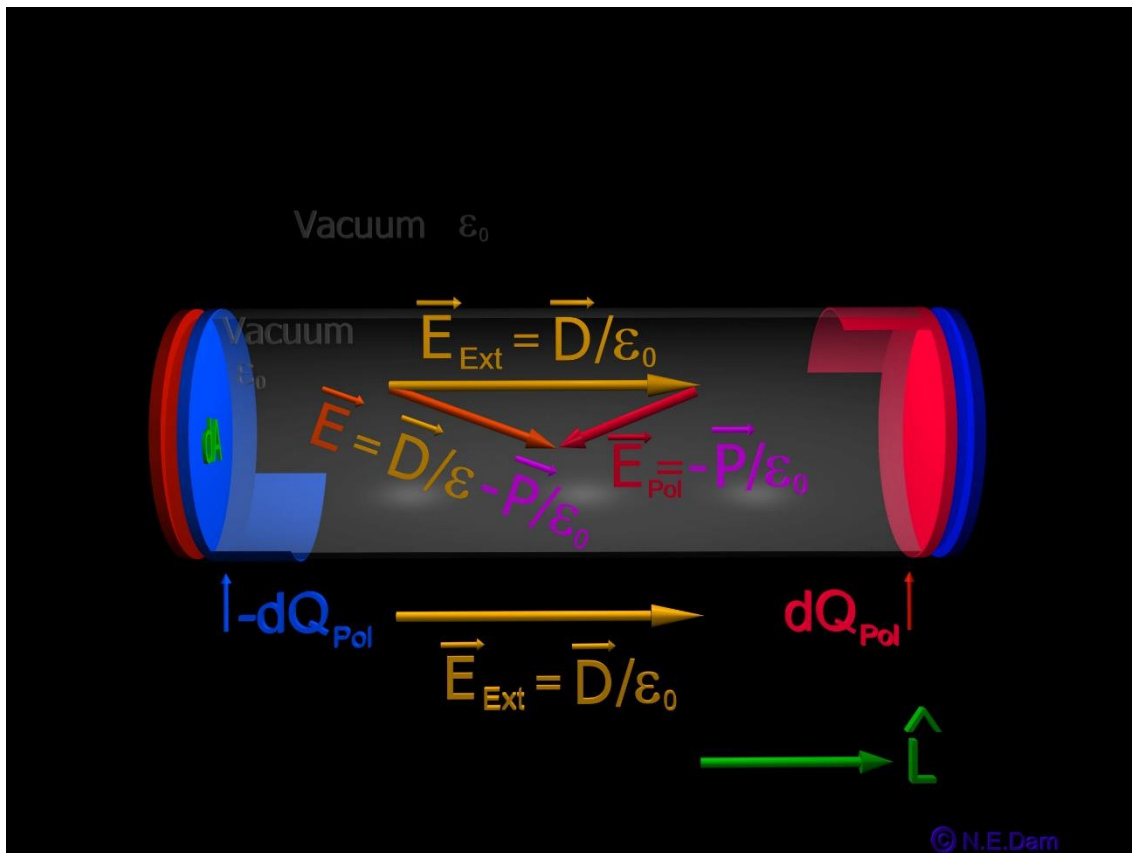


Fig.7.6.2 (N.E. Dam ©). For an anisotropic dielectric material, the vectors $\vec{E}_{\text{Ext}}$ and $\vec{E}_{\text{Pol}}$ are not parallel.

7.7 Isotropic, linearly dielectric materials

Such a material is also called a *simple dielectric material*. The word *isotropic* means, that the dielectric polarization $\vec{P}$ has the same direction as the total E-field $\vec{E}$, see fig.7.5.1. The word *linearly* means, that the polarization $\vec{P}$ is proportional with $\vec{E}$:

$$\frac{\vec{P}}{\epsilon_0} \equiv \chi \vec{E} \quad \text{eq. 7.7.1}$$

The electric susceptibility χ is related to the relative permittivity this way:

$$1 + \chi \equiv \epsilon_r \quad \text{eq. 7.7.2}$$

We repeat, that

- ϵ_0 is the permittivity for vacuum
- ϵ_r is the permittivity relative to vacuum (or the dielectric constant)
- ϵ is the absolute permittivity for a dielectric material, $\epsilon = \epsilon_r \epsilon_0$ eq. 7.7.3

Substituting into eq.7.5.1 we find the result for a simple dielectric material

$$\begin{aligned} \vec{E} = \vec{E}_{\text{Ext}} + \vec{E}_{\text{Pol}} &= \frac{\vec{D}}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} = \frac{\vec{D}}{\epsilon_0} - \chi \vec{E} \Rightarrow \vec{E} (1 + \chi) = \frac{\vec{D}}{\epsilon_0} \Rightarrow \\ \vec{E} &= \frac{\vec{D}}{\epsilon_0} \frac{1}{(1 + \chi)} = \frac{\vec{D}}{\epsilon_0} \frac{1}{\epsilon_r} = \frac{\vec{D}}{\epsilon} \Rightarrow \vec{E} = \frac{\vec{D}}{\epsilon} \quad \text{eq. 7.7.4} \end{aligned}$$

7.8 A double layer of two dielectric materials

The D-field

We consider a capacitor with a double layer of two different dielectric materials with different permittivities ϵ_1 and ϵ_2 , see fig.7.8.1. The layer thicknesses are d_1 and d_2 . To begin with, we remove the two layers and calculate the E-field in the empty capacitor. Using Gauss's law from chapter 3 in this situation, we may find the magnitude of the external E-field to be

$$E_{\text{Ext}} = \frac{\sigma_{\text{Ext}}}{\epsilon_0} \quad \text{eq. 7.8.1}$$

As mentioned earlier, the D-field is the $\epsilon_0 E$ -field in vacuum meaning that the D-field is the E-field from the external charges when we had removed the dielectric materials.

$$\vec{D} = \epsilon_0 \vec{E}_{\text{Ext}} \quad \text{eq. 7.8.2}$$

The D-field may be calculated as follows:

1. Remove the dielectric materials
2. Calculate the E-field
3. Multiply the E-field by ϵ_0 to get the D-field

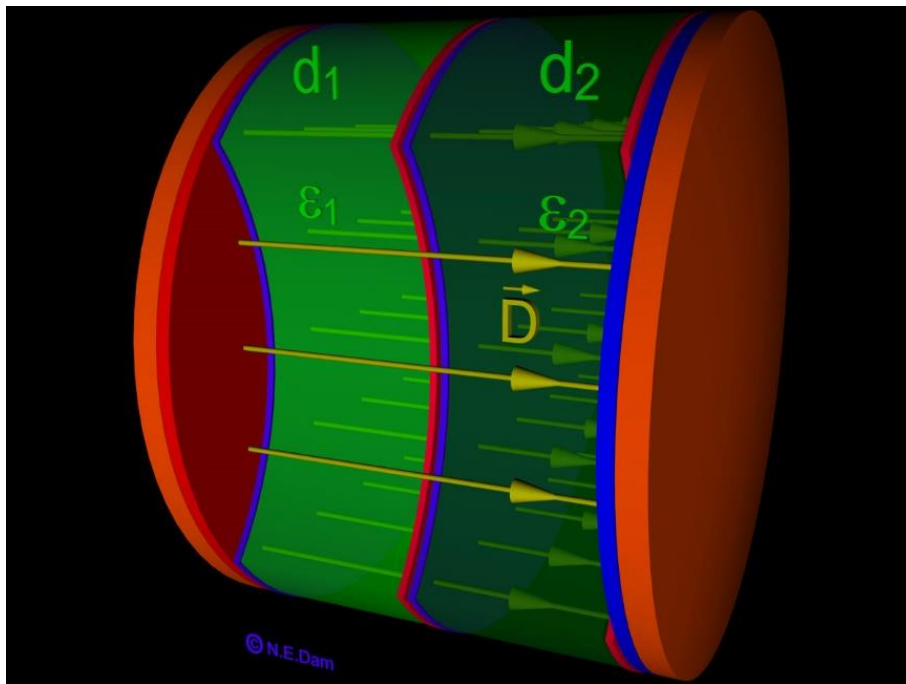


Fig.7.8.1 (N.E. Dam ©). The D-field in between the capacitor plates is not affected by the dielectric materials.

When we insert the two layers again, the D-field will be unchanged in both layers. The D-field has the same value as without the two layers. The D-field lines only start and stop on the real external charges on the capacitor plates.

However, the D-field causes the two layers to be polarized. Each of the two layers develop an end surface with positive polarization charge and an end surface with negative polarization charge as indicated in the figure.

The E-field in the two layers

The E-field is different in the two layers and also different from what it would have been without the dielectric materials. We assume, that we have isotropic, linearly dielectric materials as described in section 7.7. The E-field may then be calculated from

$$\vec{E} = \frac{\vec{D}}{\epsilon} \quad \text{eq. 7.7.4}$$

In the present geometry with isotropic materials, all fields are perpendicular to the capacitor plates and we may simplify eq.7.7.4 to

$$E = \frac{D}{\epsilon} \quad \text{eq. 7.8.3}$$

For the two layers we find

$$E_1 = \frac{D}{\epsilon_1}, \quad E_2 = \frac{D}{\epsilon_2} \quad \text{eq. 7.8.4}$$

The D-field is the same in the two equations. Hence, we may find the E-field in a layer as the D-field divided by the absolute permittivity of the present layer. It is a big advantage of the D-field, that it is easy to calculate the D-field and then simply divide the D-field by the permittivities in the layers to get the E-fields in these layers. It would have been much more complicated to calculate these E-fields without using the D-field.

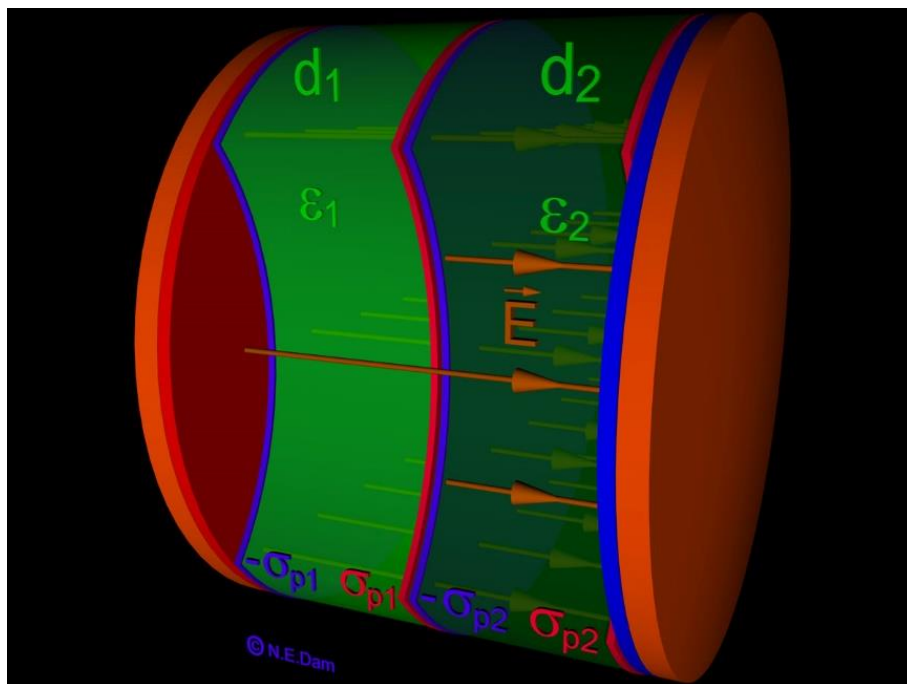


Fig.7.8.2 (N.E. Dam ©). The E-field in the two layers may be calculated from the D-field and the permittivity in the two layers.

The change in the dielectric polarization

We repeat the definition of the dielectric polarization from section 7.4 and fig.7.4.1:

The dielectric polarization $\vec{P}$ is the electric dipole moment per volume

$$\vec{P} \equiv \frac{d\vec{p}_E}{dV} = \frac{dQ_{pol}\vec{L}}{dA L} = \sigma_{pol}\hat{L} \quad \text{eq. 7.4.6}$$

The physical meaning of the dielectric polarization vector $\vec{P}$ is :

1. *The direction is the displacement direction of the positive polarization charge*
2. *The magnitude is the amount of displaced polarization charge per area, where the area is perpendicular to the displacement direction*

In fig.7.8.3 the dielectric polarization $\vec{P}_1$ represents positive polarization charge, which has been displaced to the right because of the D-field, which is to the right. Hereby, positive polarization charge is accumulated at the right end surface of layer number 1. The density of this polarization charge is

$$P_1 = \sigma_{p1} \quad \text{eq. 7.8.5}$$

In layer number 2 we have the same situation, where positive polarization charge is displaced to the right. It also means that positive polarization charge is displaced away from the left end surface of layer number 2 as indicated by $\vec{P}_2$. Equivalently, one could say that negative polarization charge is displaced to the left towards this end surface. At the right end surface of layer 2, we have this density of polarization charge:

$$P_2 = \sigma_{p2} \quad \text{eq. 7.8.6}$$

At the left end surface of layer 2, we have this density of polarization charge:

$$-P_2 = -\sigma_{p2} \quad \text{eq. 7.8.7}$$

The total density of polarization charge in the border between the two layers is

$$\sigma_{p1} - \sigma_{p2} = P_1 - P_2 = -\Delta P \quad \text{eq. 7.8.8}$$

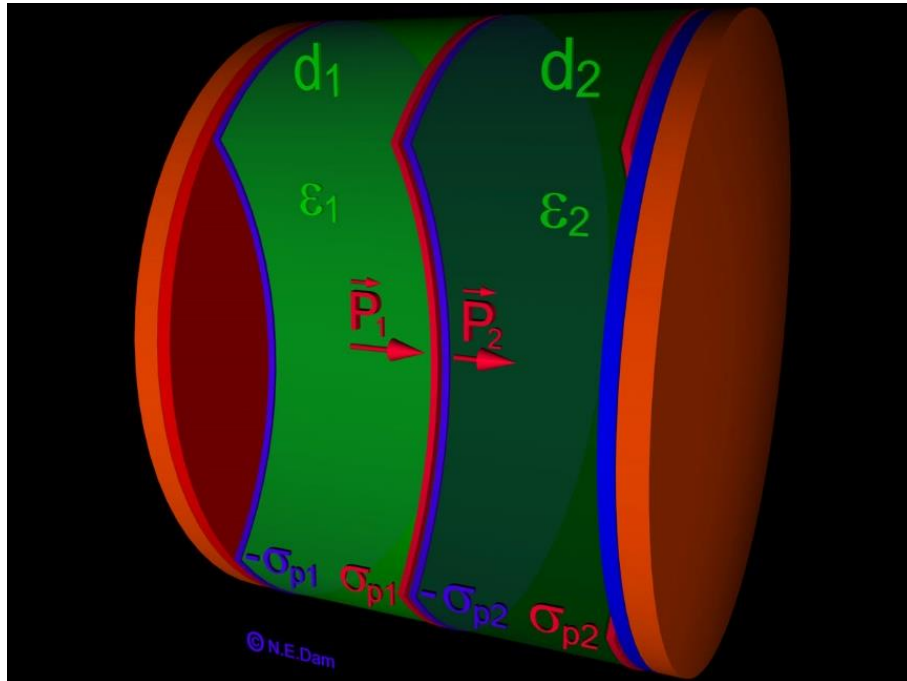


Fig.7.8.3 (N.E. Dam ©). The change in polarization may be used to calculate the density of polarization charge in the border between the two layers.

7.9 Examples of polarized molecules

Polarization of an H_2 molecule

Fig.7.9.2 shows a hydrogen molecule (H_2). It includes two protons and two electrons orbiting around in almost the same orbit. In this figure we have no external E-field from any capacitor plates. The center of the negative charge from the two electrons is in the center of the figure. Likewise, the center of the positive charge from the two protons is in the center of the figure.

Fig.7.9.3 shows the H_2 molecule when we apply an external E-field. Now, because of the electric force on the electrons, the electrons will be pulled a little to the left. Hence, the center of the negative charge from the two electrons is displaced to the left as indicated by $-q$ in the figure. Likewise, the center of the positive charge from the two protons is displaced to the right as indicated by q in the figure. An electric dipole has been created between $-q$ and q , the center for negative and positive charge.

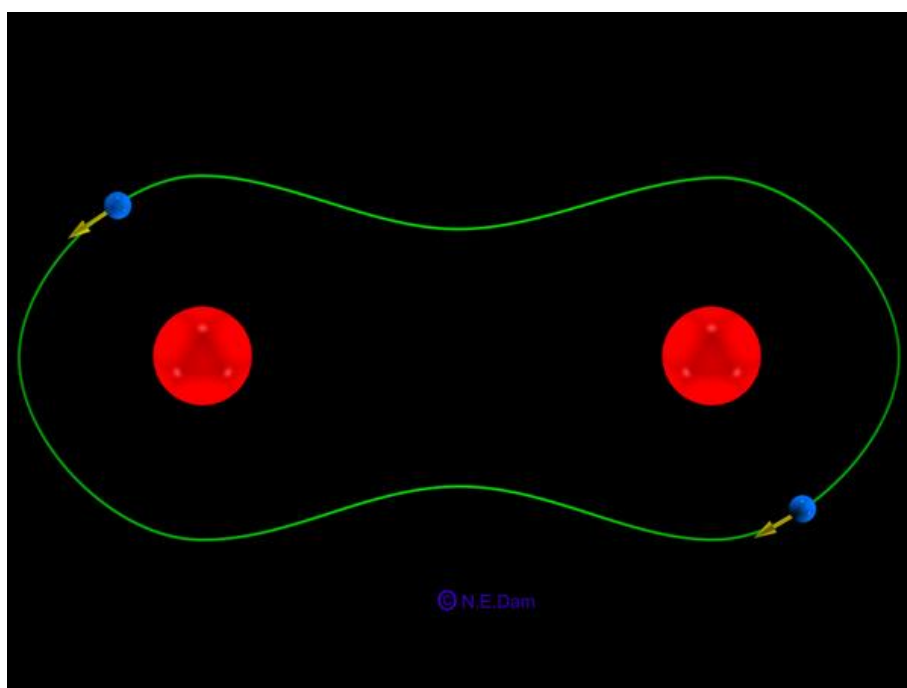


Fig.7.9.2 (N.E. Dam ©). A H_2 molecule includes two protons and two electrons. There is no external E-field.

We should remember that at room temperature, H_2 is a gas, where the molecules are moving very rapidly in straight lined zig-zag orbits. Hence a container filled with H_2 will not develop a positive and a negative side like in a solid matter. However, the explanation above shows how a molecule may develop a dipole.

The polarization in a gas is very small, but it exist and increases if the gas is cooled down and becomes a liquid.

Plastic molecules are often very long molecules consisting of long chains of Carbon atoms (C) surrounded by Hydrogen atoms (H). The long $-CH_2-CH_2-$ chains are – in principle – polarized in the same way as a simple H_2 molecule.

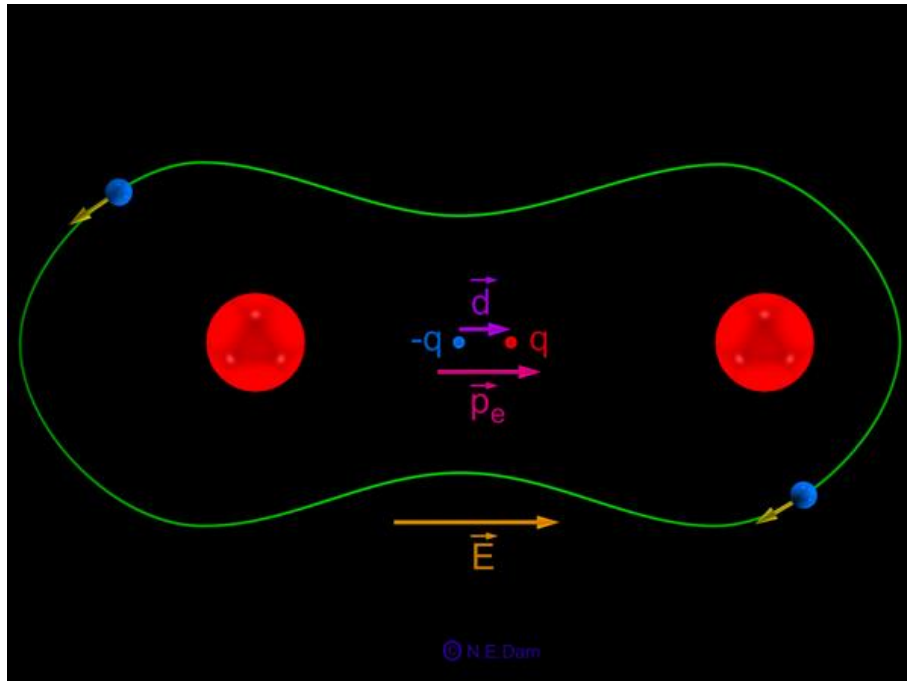


Fig.7.9.3 (N.E. Dam ©). The H_2 molecule when we apply an external E -field.

Polarization of an ionic crystal (NaCl structure)

The NaCl-structure in fig.7.9.6 is a superposition of the two structures shown in fig.7.9.4 and fig.7.9.5.

The structure in fig.7.9.4 is a so-called unit cell of Na^+ ions. There is an ion in each corner of a cube. There are 8 corners. The green lines indicate the *cubic structure*. In the real world, the green lines are – of course – not there and the red spheres are larger. We also see, that there is an extra ion in the middle of each of the 6 faces indicated by the yellow lines. Therefore the structure is called a FCC, Face Centred Cubic structure. The dimension of the cube is given by the length of one of the green lines between the center of two neighbour ions placed at the corners. This dimension is also known as the lattice constant a_0 .

Fig.7.9.5 shows the same FCC structure for the larger Cl^- ions. The cube in the structure has the same lattice constant as the cube with the Na^+ ions.

The FCC structures in fig.7.9.4 and fig.7.9.5 would not be possible in the real world because of the strong repulsive forces. However, if we combine the two structures as shown in fig.7.9.6 we get a very strong structure with high binding energies between the ions, because all ions are surrounded by ions of the opposite kind. The binding between a Na^+ and a Cl^- ion is explained in section 4.3.

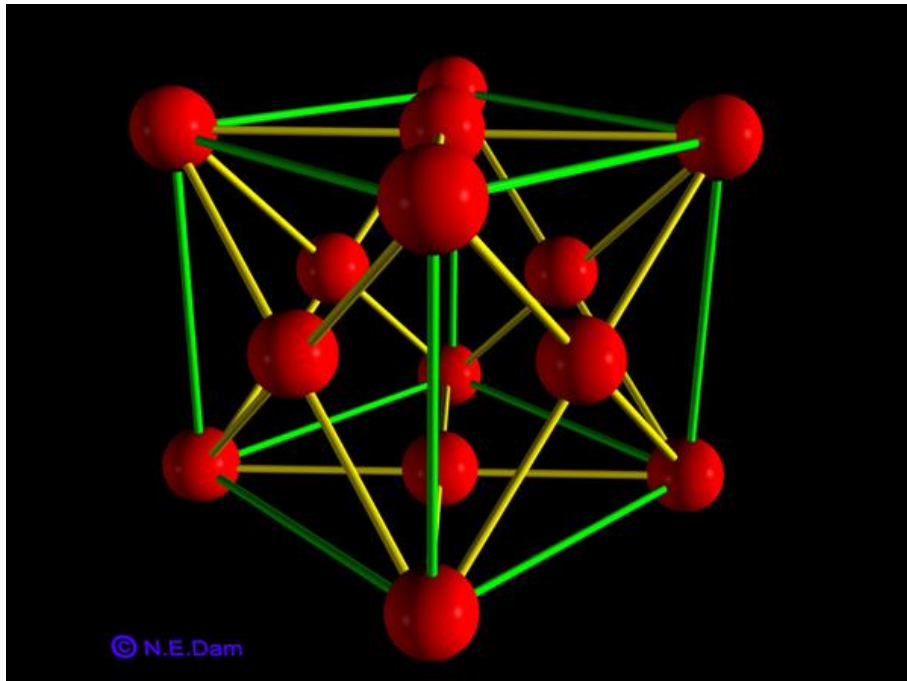


Fig.7.9.4 (N.E. Dam ©). The red spheres are the Na^+ ions. They form a so-called FCC, Face Centered Cubic structure.

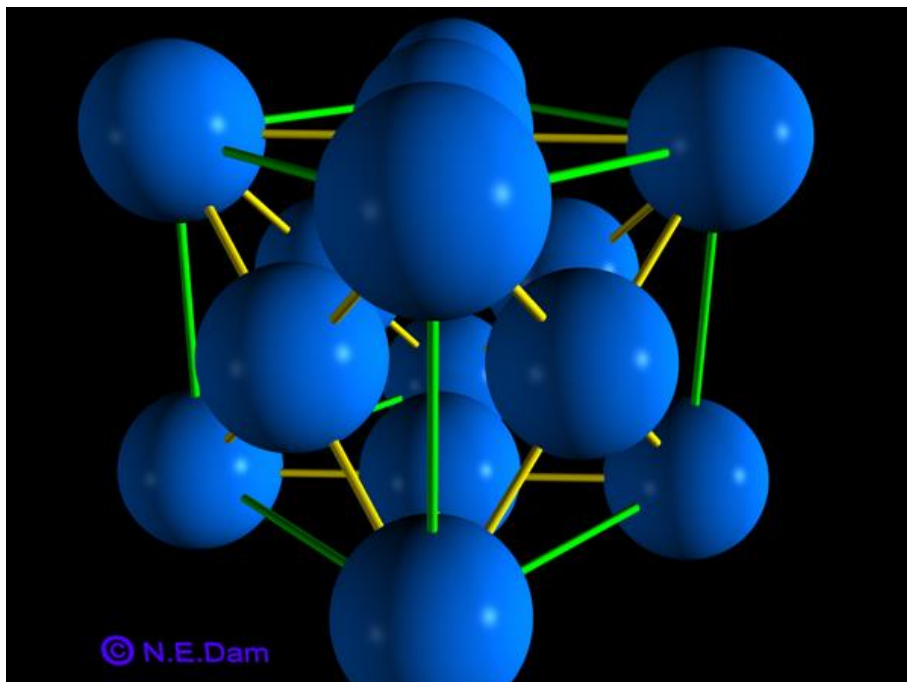


Fig.7.9.5 (N.E. Dam ©). The blue spheres are the Cl^- ions. They also form a FCC structure.

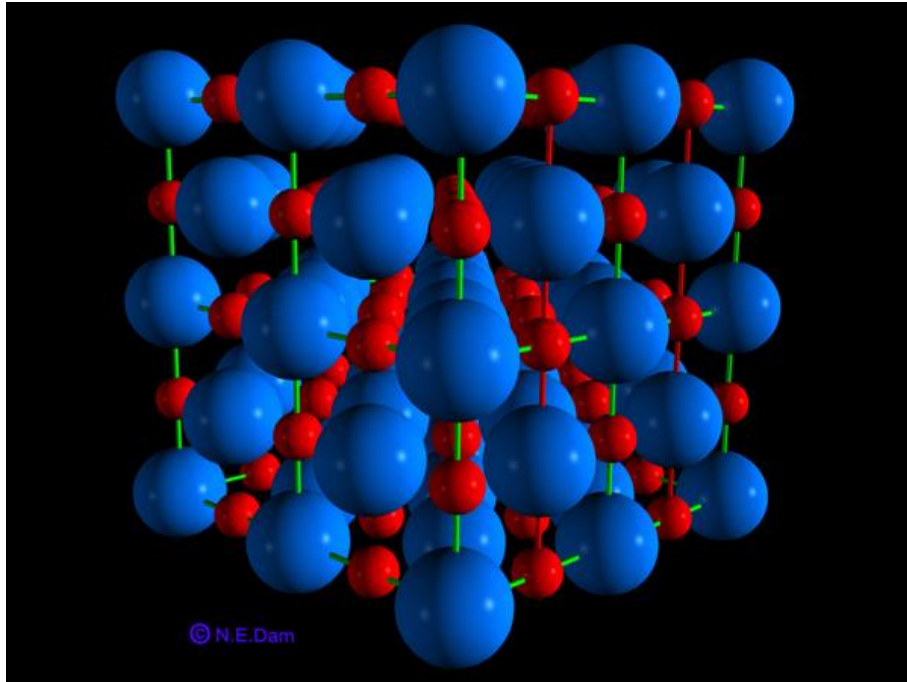


Fig.7.9.6 (N.E. Dam ©). The NaCl structure. Red spheres are the Na^+ ions, while the blue spheres are the Cl^- ions. In this figure we have no external E-field.

Remarks about the Na^+ and Cl^- ions

The red spheres are the relatively small Na^+ ions and the blue spheres are the relatively large Cl^- ions. Originally the Na atom has only one electron in its outer electron shell. This electron is loosely bound to the nucleus and is transferred to the Cl atom, which has 7 electrons in its outer electron shell.

Thereby the Cl atom becomes a negative ion, Cl^- with 8 electrons in its outer electron shell, which is a very stable electron configuration with a relatively high electron binding energy. When the electron from the outer shell of the Na atom is transferred to the outer shell of the Cl atom, this electron gets a much higher binding energy. Hence, this electron “prefers” to stay at the Cl atom instead of at the Na atom.

The Na atom becomes a positive ion, Na^+ , when it loses its outer electron. Since the Na atom has only one electron in its outer electron shell, the Na atom also loses the entire outer shell, when it loses its outer electron. It explains the relatively small ionic radius of Na^+ (98 pm, ref.7) relative to the larger Cl^- ions (181 pm, ref.7), where the outer shell is filled with 8 electrons.

Fig.7.9.7 shows what happens, if we apply an E-field to the structure. The red charges, the Na^+ ions, are now displaced a little in the direction of the E-field producing a positive end surface. The Cl^- ions are displaced in the opposite direction and produce a negative end surface on the opposite side.

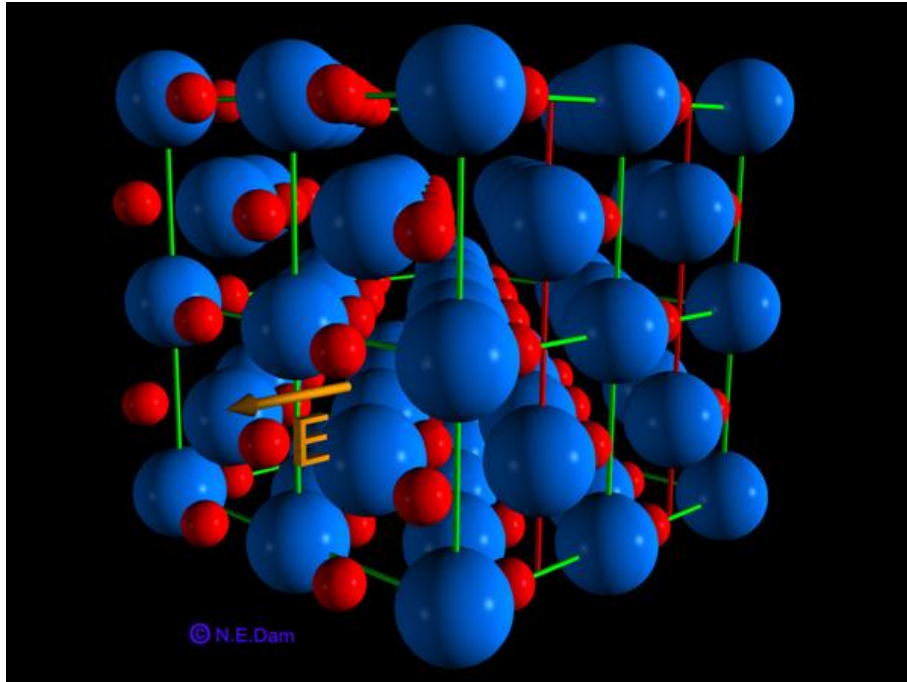


Fig.7.9.7 (N.E. Dam ©). The NaCl structure when we apply an external E-field indicated by the orange vector.

Polarization of a H₂O molecule

Fig.7.9.8 shows a H₂O molecule. The molecule itself is polarized from the beginning. Originally the O atom has 6 electrons in the outer electron shell. Therefore the O atom “wants” to get two more electrons, because this would make the outer electron shell completely filled with 8 electrons. The two extra electrons come from the two H atoms. In reality, the O²⁻ ion does not “own” the two extra electrons all the time, but most of the time. A relatively small fraction of the time, the H⁺ ions “borrow” their “own” electrons again.

Fig.7.9.8 also shows, that the center of the negative charge and the center of the positive charge are not coincident. Hence, all the water molecules are “born” as dipoles (unlike the H₂ molecules), even without an external E-field. However, if you have liquid water, the electric polarization vector $\vec{p}_e$ will point in all random directions, so the sum of all the $\vec{p}_e$ vectors will be zero.

If we apply an external E-field, as shown in fig.7.9.9, the water molecules will try to rotate, so that the $\vec{p}_e$ vectors will be parallel with the external E-field. Now, the sum of all the $\vec{p}_e$ vectors will not be zero.

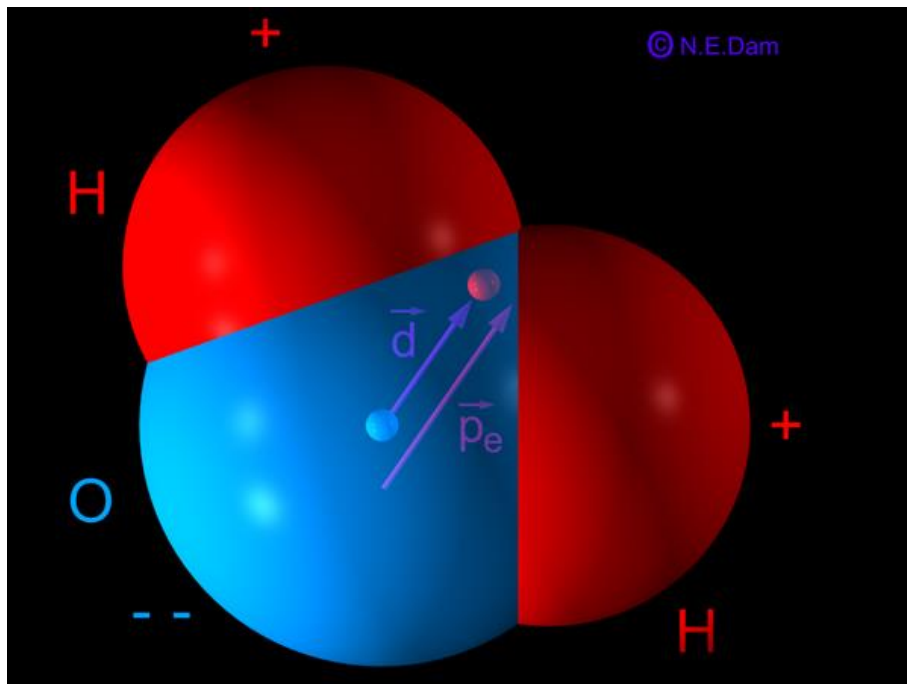


Fig.7.9.8 (N.E. Dam ©). The H_2O molecule when we have no external E -field.

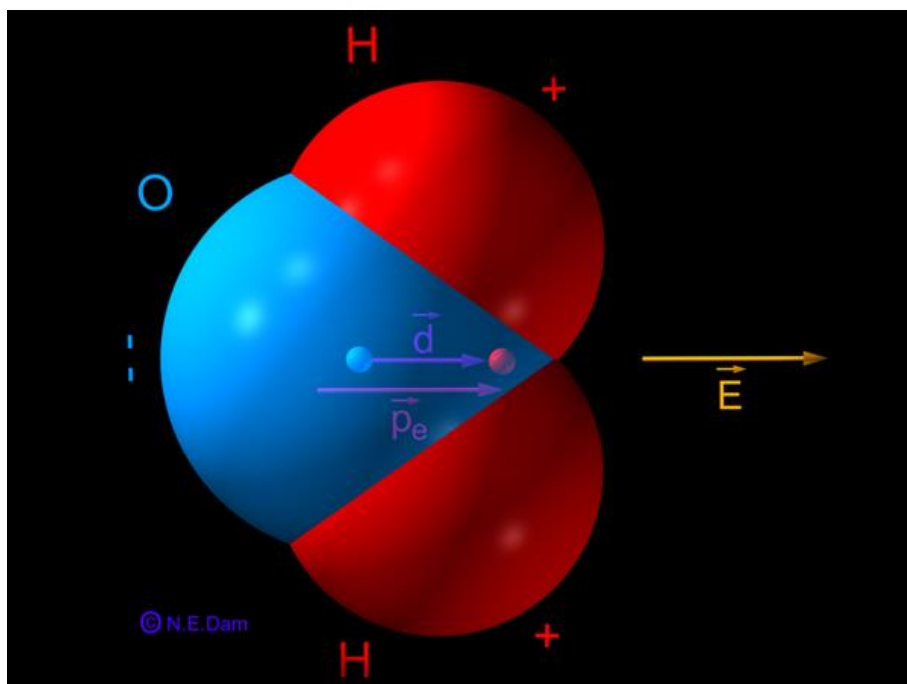


Fig.7.9.9 (N.E. Dam ©). The H_2O molecule when we have an external E -field.

The polarization in general

To summarize, the polarization may be due to three different possibilities:

1. A non-polar molecule (e.g. H_2) becomes polarized because positive charge inside each molecule is displaced in direction of $\vec{E}$ and negative charge inside each molecule is displaced in the opposite direction.

2. A crystal (e.g. NaCl) containing positive and negative ions becomes polarized because the positive ions are displaced in direction of $\vec{E}$ and negative ions are displaced in the opposite direction.
3. A polarized molecule (e.g. H₂O) is rotated so its electric polarization vector $\vec{p}_E$ becomes coparallel with $\vec{E}$.

The polarization process may be a combination of 1,2, and 3.

Any kind of polarization process may be equivalated by a displacement of positive polarization charge, normally in the same direction as $\vec{E}_{\text{Ext}}$, and a displacement of negative polarization charge in the opposite direction.

Note, that the total E-field is a combination of the external field and the polarization E-field.

$$\vec{E} = \vec{E}_{\text{Ext}} + \vec{E}_{\text{Pol}} \quad \text{eq. 7.9.1}$$

The above process 3 is illustrated in fig.7.9.10. The molecules are polarized from the beginning without an external E-field. The water molecule in section 7.9.4 would behave like this.

Without an external E-field across a liquid or a gas of water, the individual $\vec{p}_{E, \text{molecule}}$ vectors are pointing in random directions.

We equivalate a molecule by a dipole, that consists of two charges $-q_{\text{Pol}}$ and q_{Pol} with the internal distance $\vec{d}$. The electric dipole moment for a single molecule is defined by

$$\vec{p}_{E, \text{molecule}} = q_{\text{Pol}} \vec{d} \quad \text{eq. 7.4.1}$$

We can apply an external E-field across the material if we place the material in between charged capacitor plates.

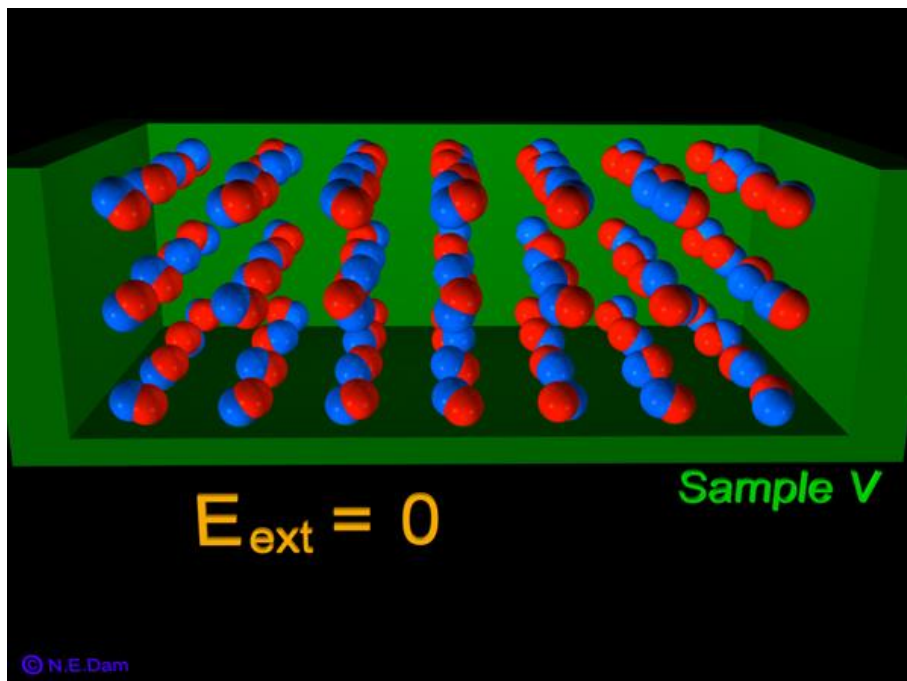


Fig.7.9.10 (N.E. Dam ©). The individual $\vec{p}_{E, \text{molecule}}$ vectors are in general pointing in random directions without an external E-field.

Now, there will be a torque acting on each molecule as described in chapter 2 and 4. The individual molecules will then rotate and be in the same direction as the total E-field as illustrated in fig.7.9.11. Perhaps the molecules will only rotate a little, it depends on how strong the total E-field is. However, in the figure, the molecules have orientated exact after the total E-field. In this case it is in the same direction as the external E-field.

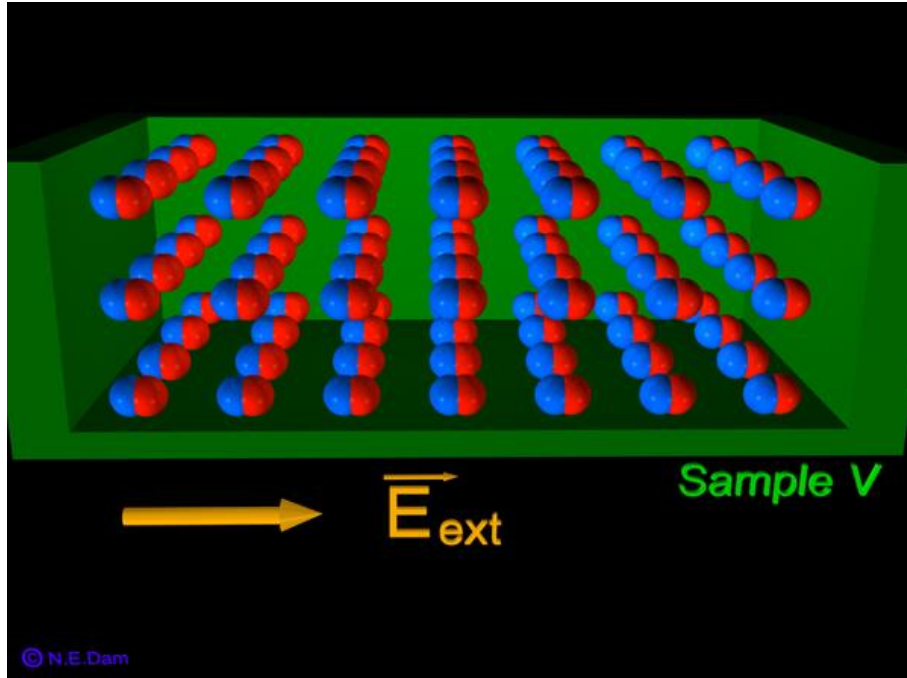


Fig.7.9.11 (N.E. Dam ©). The individual $\vec{p}_{E, \text{molecule}}$ vectors are orientated after an external E-field.

If the external E-field is increased in magnitude, the center for positive and negative charge in each molecule will be displaced more and more in opposite directions. Now, the distance $\vec{d}$ in eq.7.4.1 will increase in magnitude and we will get stronger molecular dipoles as illustrated in fig.7.9.12.

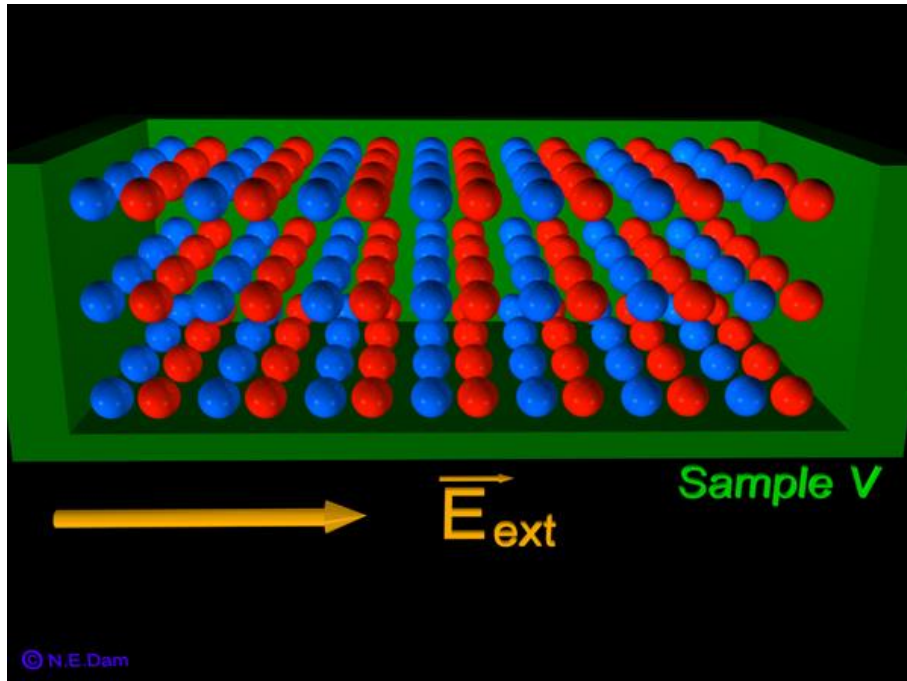


Fig.7.9.12 (N.E. Dam ©). If the external E-field increases in magnitude, there might be an increase in magnitude for the individual $\vec{p}_{E, \text{molecule}}$ vectors.

As mentioned above, the electric dipole moment for a molecule is denoted $\vec{p}_{E, \text{molecule}}$. It should be mentioned, that thermal disorder oppose the alignment of $\vec{p}_{E, \text{molecule}}$ after the E-field. At relatively low temperatures the alignment becomes better than at higher temperatures. If the alignment becomes perfect, the material becomes “saturated”.

Problems

Pr.7.1 A capacitor with a dielectric material and air

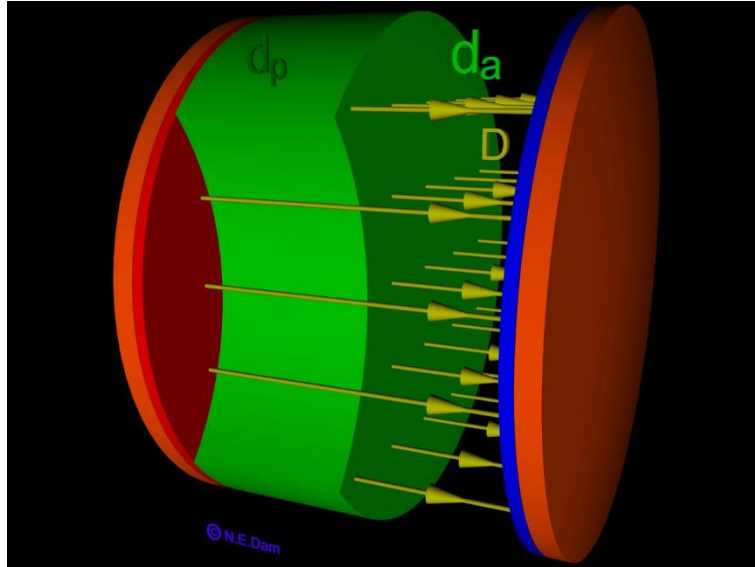


Fig.7.1a (N.E. Dam ©). A cylindrical capacitor with a plastic plate and an air gap.

This problem may be considered as a continuation of problem Pr.6.1.

The radius of the capacitor plate is R and $\epsilon_0 = 8,854 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$.

You should use your own data from Pr.6.1, otherwise use $R = 0.06 \text{ m}$ and $\epsilon_{r,p} = 2,8$

The thickness of the plastic plate is $d_p = 2 \text{ mm}$, the air gap is $d_a = 2 \text{ mm}$.

The capacitor without the plastic plate is charged by connecting it to a 3V DC voltage source and then disconnected, i.e. $V_0 = 3 \text{ V}$ and $d = d_a + d_p = 4 \text{ mm}$.

1. Find $C_0 = \epsilon_0 \frac{A}{d}$ and $Q_0 = C_0 \cdot V_0$, then find E_{Ext} . Now, we insert the plastic plate.
2. Find D_a, D_p , i.e. the D-field in the air and in the plastic.
3. Find E_a and E_p using $\frac{\vec{D}}{\epsilon} = \vec{E} \Rightarrow E = \frac{D}{\epsilon}$
4. Sketch in two profiles the D-field pattern and the E-field pattern
5. Use $\vec{E} = \vec{E}_{\text{Ext}} + \vec{E}_{\text{pol}} = \frac{\vec{D}}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} \Rightarrow E = \frac{D}{\epsilon_0} - \frac{P}{\epsilon_0}$ to show that $P = D - \epsilon_0 \cdot E$
6. Find P_a and P_p
7. Find σ_p on the plastic surfaces and show it on a sketch together with P_a and P_p .
8. Find V_a and V_p , the voltage drop across the air and the plastic and find $V = V_a + V_p$
9. Find $C = \frac{Q}{V}$ and compare with a measurement if possible.

Test 7 Dielectric materials

1. (15p). Please make a drawing, that shows:

- The two capacitor plates (Q_{Ext} and $-Q_{\text{Ext}}$) generating $\vec{E}_{\text{Ext}}$ (to the right)
- The two layers of polarization charge ($-dQ_{\text{Pol}}$ and dQ_{Pol}) in vacuum, that has replaced the sample (volume $dA L$) and generated $\vec{E}_{\text{Pol}}$ to the left
- You do not need to write formulas on the figure, just indicate the charges and the fields mentioned above and the total E-field.

2. (35p) Formulas related to the P-field

- (5p) Please write the dipole moment $\vec{p}_{\text{E, molecule}}$ for a single molecule using q_{Pol} and $\vec{d}$.
- (5p) Please write the electric dipole moment $\vec{dp}_{\text{E}}$ for a sample volume dV as a sum of the molecular dipole moments.
- (5p) Write the definition of the dielectric polarization vector $\vec{P}$ using $\vec{dp}_{\text{E}} = dQ_{\text{Pol}}\vec{L}$ and $dV = dA L$ and express $\vec{P}$ using σ_{Pol} and $\hat{L}$
- (10p) Explain the physical meaning of the direction and magnitude of vector $\vec{P}$.
- (5p) Please write E_{Pol} using σ_{Pol} and ϵ_0 .
- (5p) Write the relation between $\vec{E}_{\text{Pol}}$ and $\vec{P}$

3. (20p) The D-field

- (10p) Write the connection between the external E-field and the D-field and explain the D-field in words.
- (5p) The total E-field as a vectorsum of the external and the polarization E-field
- (5p) Write the magnitude of the total E-field in case the external and the polarization E-fields are antiparallel.

4. (30p) Isotropic, linearly dielectric material

- (5p) What does isotropic means?
- (5p) What does linearly means?
- (5p) How to express, that $\frac{\vec{P}}{\epsilon_0}$ is proportional with the total E-field $\vec{E}$
- (5p) The name and the symbol for the proportionality constant
- (10p) Derive the relation between $\vec{E}$ and $\vec{D}$ in such a material.