

Electrophysics 1

N E Dam

December 5, 2023

Contents

9	Semiconducting materials	5
9.1	Intrinsic semiconductors	7
9.1.1	The crystal structure of Si and Ge	7
9.1.2	Thermal excited electrons and holes	9
9.1.3	Drift velocities of holes and electrons in an intrinsic semiconductor .	11
9.1.4	The concentration of holes and electrons in an intrinsic semiconductor	13
9.1.5	The current density and conductivity of an intrinsic semiconductor .	14
9.2	N-type semiconductor	16
9.3	P-type semiconductor	18
9.4	Drift current and diffusion current	21
9.5	The PN-junction without a current	24
9.6	Profile of the volume charge density	29
9.7	Concentration profiles	31
9.8	E-field profile	34
9.9	Potential profile	37

Chapter 9

Semiconducting materials

Introduction

Semiconductor materials are essential to all of modern civilization. Diodes, transistors and many other semiconductor components are included in all kinds of electronics, smart-phones, power regulation equipment and much more. This chapter deals with the pure I-type semiconductors as well as doped P-type and N-type semiconductors with an excess of positive and negative charge carriers, respectively. A diode is made by combining a P-type and an N-type semiconductor, creating a so-called PN junction. We derive profiles for charge density, electric field and electric potential through this PN junction to achieve a deep understanding of how a diode works. This forms the basis for further studies of all other semiconductor components.

Before you read this chapter it is recommended to read the chapters 2, 3, 4, 5, 8. Here is a brief overview of the following sections:

1. Section 9.1 describes electric conduction in pure semiconductors, also known as intrinsic semiconductors or I-type semiconductors.
2. Section 9.2 describes electric conduction in N-type semiconductors, where conduction electrons are the dominating particles transporting charge through the material.
3. Section 9.3 describes electric conduction in P-type semiconductors, where positively charged holes are the dominating “particles” transporting charge through the material.
4. Section 9.4 describes the two kinds of currents, drift current and diffusion current, which are important for the function of a PN-junction.
5. Section 9.5 describes the formation of a PN-junction when a P-type semiconductor is connected to an N-type semiconductor. The PN-junction is the central part of a

diode. The migration of conduction electrons and holes across the border between the two semiconductors is described without having an external electrostatic potential across the PN-junction. We consider the concentration profiles of conduction electrons and holes across the border. Thereafter, we use the profile of the volume charge density to calculate the profile of the E-field and the electrostatic potential across the border.

6. Section 9.6 describes a PN-junction when an external electrostatic potential is applied. Here, we see the reason why the PN-junction is able to open for a current in one direction, but not in the opposite direction.

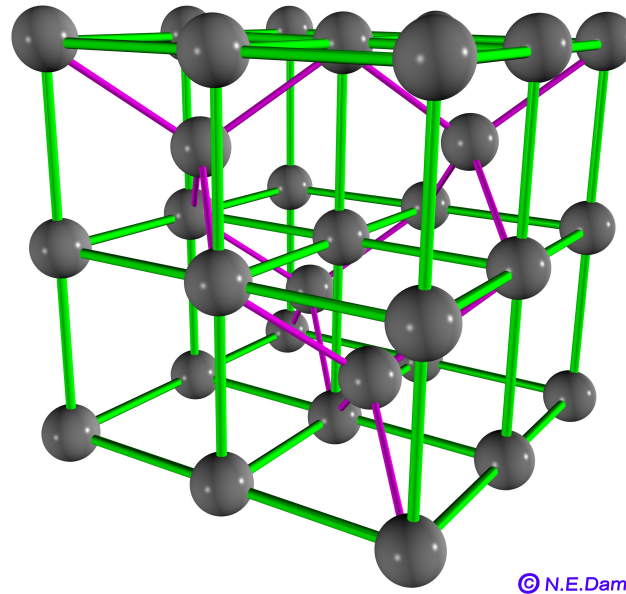


Figure 9.1. A unit cell with the so-called diamond structure. This is the crystallographic structure of carbon (C), silicon (Si) and germanium (Ge).

9.1 Intrinsic semiconductors

9.1.1 The crystal structure of Si and Ge

An intrinsic semiconductor - or *I-type semiconductor* for short - is a completely pure semiconductor to which no "foreign" atoms have been added. If, for example, the semiconductor consists of the element silicon (Si), there are only Si atoms in the crystal structure.

The most commonly used semiconductor is silicon and this is the material used all the way through the rest of this chapter. In older days germanium (Ge) was used very much as the semiconducting material. Light emitting diodes, LED's, are typically made of gallium arsenide (GaAs) doped with the element phosphorus (P).

A unit cell is the smallest group of atoms with the same structure as a crystal. If the unit cell is repeated along the x, y, and z-directions, we get the crystal in full-size. The unit cell is like a brick in a building.

Figure 9.1 shows a unit cell with the *diamond structure*. This is the crystallographic structure of a diamond, where all the grey lattice points are occupied by carbon (C) atoms. The semiconducting materials silicon (Si) and germanium (Ge) have the same crystallographic structure as carbon (C) crystallized as a diamond. The three elements Ge, Si, and C(diamond) are said to be isotypical, because they have the same type of

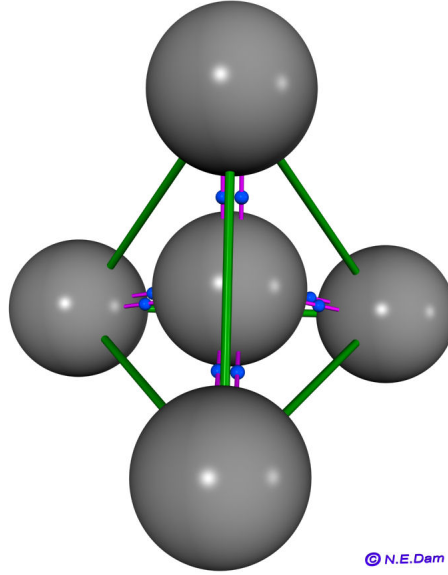


Figure 9.2. A so-called tetrahedron with 5 Si atoms is a part of the unit cell with the diamond structure shown in figure 9.1.

crystal structure. Furthermore, the diamond structure is said to be homodesmic, because there are mainly one type of bond in the structure, namely the covalent bond.

The overall structure shown in figure 9.1 is a Face Centered Cubic structure (FCC). The structure is denoted cubic because each of the 3 sides of the unit cell has the same length. The name “Face Centered” is because each of the 6 faces, has an atom in the middle. In figure 9.1 we see that the cubic unit cell may be divided into 8 smaller cubic cells, so-called cubelets. The smaller cubic cells, the cubelets, are not unit cells since they cannot be repeated to give the crystal structure. In 4 of the cubelets there is an extra atom in the centre surrounded by 8 atoms in the corners. This “centre atom” binds to 4 of the 8 “corner atoms” as indicated by the purple lines. Such a “centre atom” with bonds to 4 “corner atoms” is shown in the next figure.

Figure 9.2 shows a so-called tetrahedron with 5 Si atoms. The “centre atom” in the middle binds to 4 “corner atoms”. Each Si atom has a *coordination number* of 4 meaning, that it coordinates 4 other atoms. In other words each Si atom binds to 4 neighbours. The 4 bonds are the so-called *covalent bonds*, where each bond has electrons from both atoms on each side of the bond. If we consider one Si atom, it has 4 electrons in its outermost shell and these 4 electrons are used in the 4 bonds, one electron per bond. Such an electron is denoted a *covalent electron*. Each covalent bond has 2 covalent electrons, one electron from each of the two atoms that are held together by the bond.

The tetrahedron is also denoted a trigonal pyramid. Such a pyramid has 4 faces in

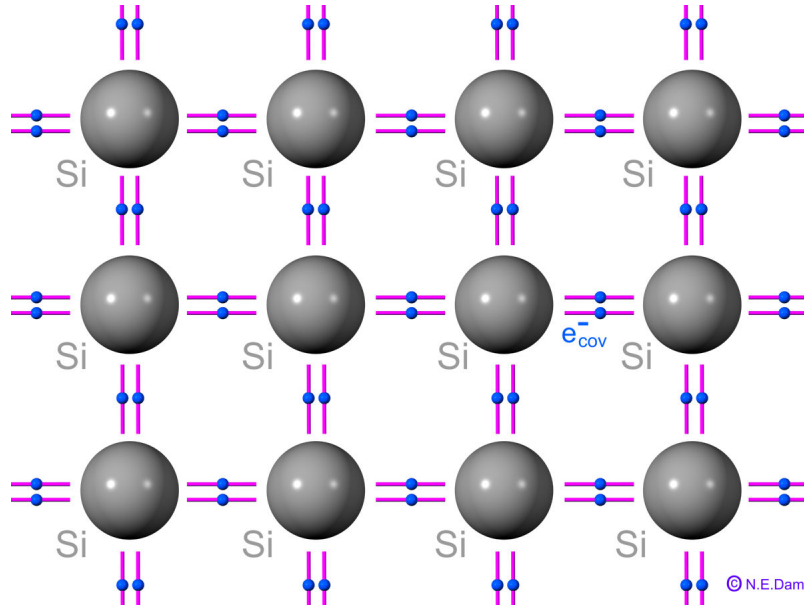


Figure 9.3. A simplified drawing of the Si crystal structure.

total, one face “facing downwards” and 3 faces “facing upwards”. In a real crystal, the Si atom in the middle “touches” the 4 neighbour Si atoms. However, the atoms in the figure are smaller in order to show the structure and the bonds more clearly.

It is difficult to draw a 3D sketch of the tetrahedrons in the Si crystal structure. Therefore, we sketch the structure in 2D as shown in figure 9.3. This sketch has also 4 bonds from each atom. All the electrons shown in the figure are covalent electrons (e_{cov}^-) forming the covalent bonds.

9.1.2 Thermal excited electrons and holes

The Si crystal structure shown in figure 9.3 is not conducting at all. At $T = 0$ K there are no conduction electrons to transport electric charge through the crystal.

However, because of thermal vibrations at $T > 0$ K in the atomic crystal lattice, energy may be transferred to one of the covalent electrons. Such an electron may be excited to the conduction band, i.e. the electron is lifted into a high energy level in the so-called conduction band, which are some very closely spaced energy levels, where the electrons can surf through the entire crystal structure and therefore conduct electric current, hence the name conduction band.

Figure 9.4 shows a pure (intrinsic) Si semiconductor, where one of the covalent electrons is excited to the conduction band. The deficit of one electron in the covalent bond is referred to as a *hole*. It is positively charged and it is denoted e_{hole}^+ . When we remove

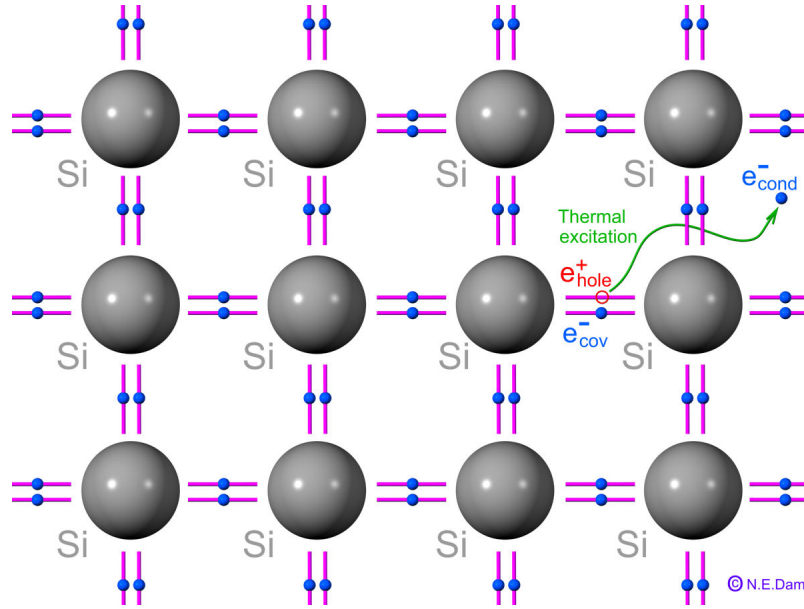


Figure 9.4. A pure (intrinsic) Si semiconductor, where one of the covalent electrons is excited to the conduction band.

one electron from an atom, the number of positive protons in the nucleus will be one more than the number of negative electrons in the shells around the atom. This explains the positive charge associated with the hole. As mentioned, a covalent electron is excited to the conduction band and becomes a conduction electron. At the same time a hole is formed. Hence, the number of conduction electrons equals the number of holes. The concentration is the number of particles per unit of volume (m^3). Therefore, the concentration of conduction electrons equals the concentration of holes. The thermal excited electrons and holes recombine all the time with the same speed as they are formed. However, there will be a certain equilibrium concentration of thermal excited electrons and holes and this concentration depends of the temperature. When the temperature T increases, the lattice vibrations become more intense and more pairs of thermal excited electrons and holes are produced. This increases the conductivity. On an energy scale of an electron, there is a so-called *energy gap* U_g from the level of a covalent electron to the level of an electron in the conduction band. E.g., at $T = 300$ K, it takes about $U_g = 1,1$ eV to excite an electron from a covalent bond to the conduction band in Si. Here, we use the energy unit eV (electron Volt) for atomic particles defined in equation 9.1.

$$1 \text{ eV} = 1,602 \cdot 10^{-19} \text{ J} \quad (9.1)$$

The charge quantum is the smallest possible amount of charge defined in equation 9.2

$$1\ e = 1,602 \cdot 10^{-19} C \quad (9.2)$$

The *concentrations of conduction electrons and holes* are defined as follows:

- n_{nI} = The number of negatively charged conduction electrons per m^3 of the Intrinsic material. NB! Note that n in the subscript is used for negatively charged conduction electrons and subscript I means Intrinsic semiconductor.
- n_{pI} = The number of positively charged holes per m^3 of the Intrinsic material. NB! Note that p in the subscript is used for positively charged holes and subscript I means Intrinsic semiconductor.

Both the negatively charged conduction electrons and the positively charged holes may transport electric charge through the crystal lattice. It means that both kinds of particles contribute to the conduction. In solid state physics one consider the hole to have the charge $+e$ and the same mass as an electron. When a hole is moving, some electrons are moving around the hole in the opposite direction. However, we do not consider such electrons, and they should not be confused with the conduction electrons coming from thermal excitations.

NB! A hole should not be confused with a positron, which is a particle with the charge $+e$ and with the same mass as an electron. Positrons exist neither in conductors nor in semiconductors. The positron occurs in connection with nuclear reactions and usually has an extremely short lifetime.

9.1.3 Drift velocities of holes and electrons in an intrinsic semiconductor

Figure 9.5 shows how the current density J is composed of the following two types of drift velocities corresponding to the two types of charges.

- v_d = drift velocity in general.
- v_n = drift velocity of negatively charged conduction electrons moving in the opposite direction of the E -field.
- v_p = drift velocity of positively charged holes moving in the same direction as the E -field.

We have the following relation between the electric E -field and the electric force acting on a particle with charge q

$$\vec{F}_e = q\vec{E} \quad (9.3)$$

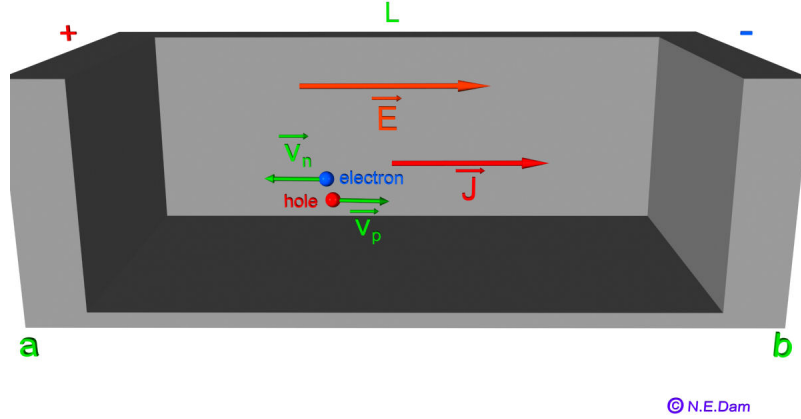


Figure 9.5. The grey material is supposed to be an intrinsic semiconductor. The electrons are moving in the opposite direction of the E -field, the holes are moving in the same direction as the E -field.

A hole has $q = e$, which is positive. Hence, the force $\vec{F}_e$ has the same direction as the E -field $\vec{E}$. Therefore, the drift velocity $\vec{v}_p$ is co-parallel with $\vec{E}$. This gives a positive contribution to the current in the right direction in figure 9.5.

An electron has $q = -e$, which is negative. Hence, the force $\vec{F}_e$ is opposite directed relative to the E -field $\vec{E}$. Therefore, the drift velocity $\vec{v}_n$ is anti-parallel with $\vec{E}$. The current is defined as the charge dQ that flows through an imaginary area A during the time dt . The area A is typically the cross section area of a wire. The current is not a vector, since it is an amount of charge per time dt , that flows through an area.

$$I \equiv \frac{dQ}{dt} \quad (9.4)$$

The current density is the current per area, where the area is perpendicular to the current

$$J \equiv \frac{I}{A} \quad (9.5)$$

In chapter 8, the current density J (but not the current I) is given a direction, that shows the direction of the movement for the positive charge. In chapter 8 we also derive the following relation between the current density vector $\vec{J}$ and the drift velocity $\vec{v}_d$ for a

charge q

$$\vec{J} = n q \vec{v}_d \quad (9.6)$$

Here n is the number of particles per m^3 . Each particle has the charge q . Note that $\vec{J}$ and $\vec{E}$ always have the same direction.

For a hole we have that q is positive and the drift velocity is denoted $\vec{v}_p$ instead of $\vec{v}_d$. Hence, $\vec{J}$ is co-parallel with $\vec{v}_p$ which is to the right in figure 9.5.

For an electron we have that q is negative and the drift velocity is denoted $\vec{v}_n$ instead of $\vec{v}_d$. Hence, $\vec{J}$ is anti-parallel with $\vec{v}_n$. Since $\vec{v}_n$ is to the left in figure 9.5, $\vec{J}$ is to the right in figure 9.5. Negative electrons moving to the left is equivalent with positive charges moving to the right.

In Chapter 8 we also present Ohm's law expressed in terms of the electric field $\vec{E}$ and the current density $\vec{J}$

$$\vec{E} = \rho \vec{J} \quad (9.7)$$

As mentioned above, the electric field $\vec{E}$ and the current density $\vec{J}$ are co-parallel. Equation 9.7 is denoted Ohm's law if ρ is a constant. This constant is called the resistivity. If we change to a different material or a different temperature, we get a different value for ρ . However, for a given material and a given temperature, ρ in equation 9.7 is constant and when Ohm's law is fulfilled. Sometimes it is more convenient to use the conductivity defined by

$$\sigma = \frac{1}{\rho} \quad (9.8)$$

In this case, Ohm's law is expressed by

$$\vec{J} = \sigma \vec{E} \quad (9.9)$$

The vectors $\vec{E}$ and $\vec{J}$ are co-parallel vectors in an electrically isotropic material, where we have the same conductivity in all directions. This is assumed to apply here. Thereby, Ohm's law can also be expressed by the lengths of the two vectors $\vec{E}$ and $\vec{J}$

$$\sigma = \frac{J}{E} \quad (9.10)$$

9.1.4 The concentration of holes and electrons in an intrinsic semiconductor

As mentioned earlier a thermal excitation produces a hole and an electron, which is excited to the conduction band. Hence, we have the same concentration of holes and thermal excited electrons

$$n_{pI} = n_{nI} \quad (9.11)$$

It is a rather long proof to show that

$$n_{pI} \cdot n_{nI} = n_{nI}^2 = K T^3 e^{-\frac{U_g}{k_B T}} \quad (9.12)$$

Here, K is a material constant, T is the absolute Kelvin temperature, U_g is the so-called *energy gap*, which is the energy gap from the energy level of a covalent electron to the energy level of the conduction band. U_g is the energy that should be supplied to excite a covalent electron to the conduction band. Finally, k_B is the Boltzmann's constant

$$k_B = 8,617 \cdot 10^{-5} eV/K \quad (9.13)$$

At $T = 300$ K we have

$$k_B T = 25,85 \cdot 10^{-3} eV \cong 26 \text{ meV} \quad (9.14)$$

9.1.5 The current density and conductivity of an intrinsic semiconductor

As mentioned earlier we have thermal excited holes and electrons in equal amounts. Both these holes and these electrons contribute to the current density. We now use equation 9.6 to calculate this current density and we consider the following two contributions

1. Contribution to the current density from thermal excited holes

The charge of a hole is $q = e$ (positive charge quantum). The drift velocity $\vec{v}_d$ is denoted $\vec{v}_p$, where p denotes the positive charge carrier (holes). According to equation 9.6 $\vec{v}_d$ is proportional to $\vec{J}$. According to equation 9.7 $\vec{J}$ is proportional to $\vec{E}$. Hence, $\vec{v}_d$ is proportional to $\vec{E}$. If we consider a hole, this may be written this way

$$\vec{v}_p = \mu_p \vec{E} \quad (9.15)$$

The proportional constant μ_p is known as the mobility of holes. We find

$$\vec{J} = n_{pI} e \vec{v}_p = n_{pI} e \mu_p \vec{E} = n_{pI} \mu_p e \vec{E} \quad (9.16)$$

2. Contribution to the current density from thermal excited electrons

The charge of an electron is $q = -e$ (negative charge quantum). The drift velocity $\vec{v}_d$ is denoted $\vec{v}_n$ (n denotes the negative charge carrier: electron) The drift velocity must be proportional with the electric field by the same reason as for the holes. However, the drift velocity $\vec{v}_n$ is anti-parallel with the electric field $\vec{E}$. Therefore, we find

$$\vec{v}_n = -\mu_n \vec{E} \quad (9.17)$$

The proportional constant μ_n is known as the mobility of electrons. We find

$$\vec{J} = n_{nI} (-e) \vec{v}_n = n_{nI} (-e) (-\mu_n \vec{E}) = n_{nI} \mu_n e \vec{E} \quad (9.18)$$

3. The total current density from thermal excited holes and electrons

The total current density becomes

$$\vec{J} = n_{pI} \mu_p e \vec{E} + n_{nI} \mu_n e \vec{E} = (n_{pI} \mu_p + n_{nI} \mu_n) e \vec{E} \quad (9.19)$$

As previously mentioned, the two vectors $\vec{E}$ and $\vec{J}$ are co-parallel vectors in an electrically isotropic material. Consequently, we can omit the vector arrows

$$J = (n_{pI} \mu_p + n_{nI} \mu_n) e E \quad (9.20)$$

According to Ohm's law expressed by equation 9.10 we find

$$\sigma = \frac{J}{E} = (n_{pI} \mu_p + n_{nI} \mu_n) e \quad (9.21)$$

Some data are given in table 9.1¹. Now, it is recommended to solve Problem 9.1 or Test 9A.

Element	μ_n	μ_p	Unit	n_{nI}	n_{atoms}	Unit
Ge	0,390	0,190	$m^2/(Vs)$	$2,4 \cdot 10^{19}$	$4,4 \cdot 10^{28}$	$1/m^3$
Si	0,135	0,048	$m^2/(Vs)$	$1,45 \cdot 10^{16}$	$5,0 \cdot 10^{28}$	$1/m^3$

Table 9.1. Hole and electron mobility data for Ge and Si at $T = 300$ K.

¹Data according to E V Sørensen, Elementær Elektronik Del 1 Halvlederdioder, DTU (1975)

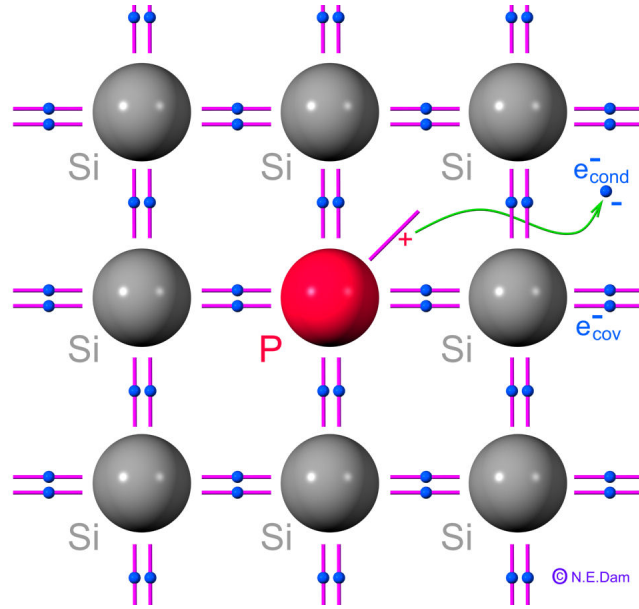


Figure 9.6. An N-type semiconductor.

9.2 N-type semiconductor

To produce an N-type semiconductor, we start with an I-type semiconductor, which is an intrinsic semiconductor. An example of such a pure semiconductor is shown in figure 9.3. Here we see a perfect pure silicon (Si) crystal structure. Then we add one phosphorus atom (P) per - let's say - one million silicon atoms, see figure 9.6. The phosphorus atom is here called the *donor atom* and the silicon crystal structure is called the *host*. The process is called *doping*.

The concentration of donor atoms (number of donor atoms per m^3) is denoted n_{Do} . The concentration of host atoms is denoted n_{Ho} . The *degree of doping* is

$$\zeta_N = \frac{n_{Do}}{n_{Ho}} \quad (9.22)$$

It is the number of donor atoms relative to the number of host atoms. It is typically in the range 10^{-4} to 10^{-7} depending on the required properties. The lower case Greek letter ζ is “zeta”.

The “impurity” atom (P) should have about the same size and polarizability² as the host atoms (Si). Thereby, P fits into the crystal structure and P can replace one of the Si atoms. However, P has 5 electrons in its outer shell while Si has 4 electrons in its outer shell. Figure 9.6 shows what happens around the P atom. The P atom in the middle of the

²the ability of being polarized

figure binds to the 4 neighbour Si atoms using only 4 of the 5 electrons in its outer shell. The fifth electron is not needed in the 4 bonds to the 4 neighbor Si atoms. It is easily excited ($U_g = 0,05 \text{ eV}$) to the conduction band where it becomes a conduction electron. The material is now denoted as an N-type semiconductor because the charge transport through the crystal lattice is mainly due to negative conduction electrons. The reason why the P atom is called a donor atom is that it delivers one electron to the conduction band. At $T = 300 \text{ K}$, it is a good approximation to say that each P atom contributes with one conduction electron. Hence, the concentration of negative conduction electrons in the N-type semiconductor (n_{nN}) is almost equal to the concentration of P donor atoms (n_{Do})

$$n_{nN} \simeq n_{Do} \quad (9.23)$$

In practice this concentration is much larger than the concentration of conduction electrons in the I-type semiconductor

$$n_{nN} \simeq n_{Do} \gg n_{nI} \quad (9.24)$$

Hence, the conductivity relative to the pure intrinsic semiconductor is greatly increased when doping the semiconductor with P. When the fifth electron from P leaves the P atom, the P atom becomes a P^+ ion with one positive charge quantum (+e). It is a good approximation to say that the fifth electron, or any other electrons, will not return to the P^+ ion. Hence, the P^+ ion will remain as a P^+ ion and this P^+ ion will stay at the same position in the crystal lattice.

In the N-type semiconductor there will also be thermal excited electrons and holes. The concentration of thermal excited electrons will be about the same as it is in an I-type semiconductor and therefore, see equation 9.24, negligible relative to the concentration of conduction electrons coming from the P donor atoms. The concentration of thermal excited holes will be even lesser than the concentration of thermal excited electrons since the thermal excited holes will be filled immediately with electrons coming from the P donor atoms. We therefore have

$$n_{pN} \ll n_{pI} \quad (9.25)$$

Ranking the concentrations we have

$$n_{pN} \ll n_{pI} = n_{nI} \ll n_{nN} \quad (9.26)$$

The relation in equation 9.12 on page 13 is also valid for an N-type semiconductor

$$n_{pN} \cdot n_{nN} = n_{nI}^2 \quad (9.27)$$

However, in this equation we have

$$n_{pN} \ll n_{nN} \quad (9.28)$$

In comparison with the I-type semiconductor, we remember equation 9.11 on page 13. Instead of phosphorus (P) one could also use **arsenic** (As) or **antimony** (Sb, latin Stibium). They also have 5 electrons in the outer shell. Now, it is recommended to solve Problem 9.2 or Test 9B.

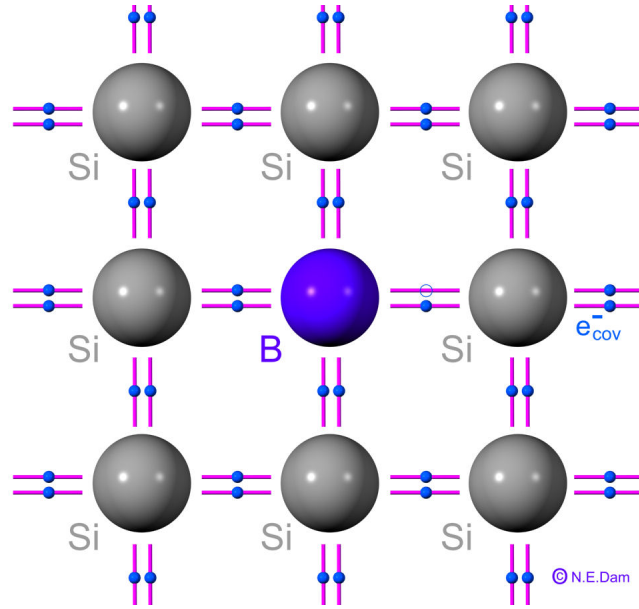


Figure 9.7. A P-type semiconductor where pure Si has been doped by B.

9.3 P-type semiconductor

To produce an P-type semiconductor, we start with an I-type semiconductor, which is an intrinsic semiconductor. An example of such a pure semiconductor is shown in figure 9.3 on page 9. Here we see a perfect pure silicon (Si) crystal structure. Then we add one boron atom (B) per - let's say - one million silicon atoms, see figure 9.7. The boron atom is here called the *acceptor atom* and the silicon crystal structure is called the *host*. The process is called *doping*.

The concentration of acceptor atoms (number of acceptor atoms per m^3) is denoted n_{Ac} . The concentration of host atoms is denoted n_{Ho} . The *degree of doping* is

$$\zeta_P = \frac{n_{Ac}}{n_{Ho}} \quad (9.29)$$

It is the number of acceptor atoms relative to the number of host atoms. It is typically in the range 10^{-4} to 10^{-7} depending on the required properties.

The “impurity” atom (B) has about the same size and polarizability as the host atoms (Si). Thereby, B fits into the crystal structure and B can replace one of the Si atoms. However, B has only 3 valence electrons in its outer shell while Si has 4 valence electrons in its outer shell.

Figure 9.7 shows what happens around the B atom. The B atom in the middle of the figure binds to the 4 neighbour Si atoms. However, one electron is missing in one of the

four covalent bonds because the B atom only contributes with 3 valence electrons. The B atom is neutral to begin with. Very quickly, the covalent bond with the missing electron becomes a complete covalent bond with 2 electrons because an electron is transferred from another covalent bond in the neighbourhood as shown in figure 9.8. The result is a negative B^- ion with 4 complete covalent bonds to the 4 neighbour Si atoms. The B^- ion remains a B^- ion because the 4 covalent bonds around the B^- ion will remain as 4 complete bonds with the same probability as if it was around a Si atom.

After a valence electron has been transferred from a covalent bond in the neighbourhood, there is a deficit of one electron in the neighbourhood as shown in figure 9.8. This deficit of one electron is denoted as a hole and the hole has one positive charge quantum because one electron is missing. This hole may be filled with an electron from another covalent bond in the neighbourhood and the process may continue. As a result, the positive hole is moving around. The material is denoted a P-type semiconductor because the charge transport through the crystal lattice is mainly due to positive holes. At $T = 300\text{ K}$, it is a good approximation to say that each B atom causes one positive hole in the crystal structure at the same time as the B atom itself becomes a stationary B^- ion in the crystal structure. Hence, the concentration of positive holes in the P-type semiconductor (n_{pP}) almost equals the concentration of B acceptor atoms (n_{Ac})

$$n_{pP} \simeq n_{Ac} \quad (9.30)$$

The reason why the B atom is called an acceptor atom is that it “accepts” one electron from the neighbourhood to complete one of its own covalent bonds. Hence, the concentration of B atoms equals the concentration of acceptor atoms (n_{Ac}).

There will be a much higher concentration of holes in the P-type semiconductor than in the intrinsic I-type semiconductor.

$$n_{pI} \ll n_{pP} \quad (9.31)$$

The intrinsic I-type semiconductor has only holes created by thermal excitation of electron-hole pairs, see figure 9.4 on page 10 and the text around this figure.

The holes in the P-type semiconductor are created in two different ways:

1. Holes caused by thermal excitation
2. Holes caused by doping with acceptor atoms

The second category dominates very much over the first category. Because of the very high concentration of holes in the P-type semiconductor, this semiconductor has much higher conductivity than the intrinsic semiconductor.

There are also thermal excited electrons in the P-type semiconductor. However, such electrons are recombined very quickly with the holes because the concentration of holes is much higher in a P-type semiconductor than in an I-type semiconductor. Hence, the

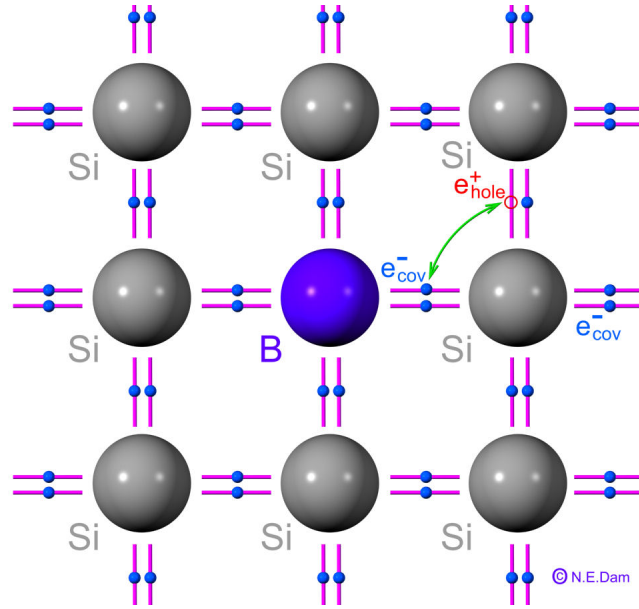


Figure 9.8. A P-type semiconductor. The covalent bond with the missing electron at the B atom becomes a complete covalent bond with 2 electrons because an electron is transferred from another covalent bond in the neighbourhood.

thermal excited electrons have a much shorter “lifetime” (and a much lower concentration) in the P-type than in the I-type semiconductor

$$n_{nP} \ll n_{nI} \quad (9.32)$$

Ranking the concentrations we have

$$n_{nP} \ll n_{nI} = n_{pI} \ll n_{pP} \quad (9.33)$$

The relation in equation 9.12 on page 13 is also valid for an N-type semiconductor

$$n_{pP} \cdot n_{nP} = n_{nI}^2 \quad (9.34)$$

However, in this equation we have

$$n_{nP} \ll n_{pP} \quad (9.35)$$

In comparison with the I-type semiconductor, we remember equation 9.11 on page 13.

Instead of boron (B) one could also use aluminium (Al), gallium (Ga) or indium (In). They also have 3 valence electrons in their outer shell.

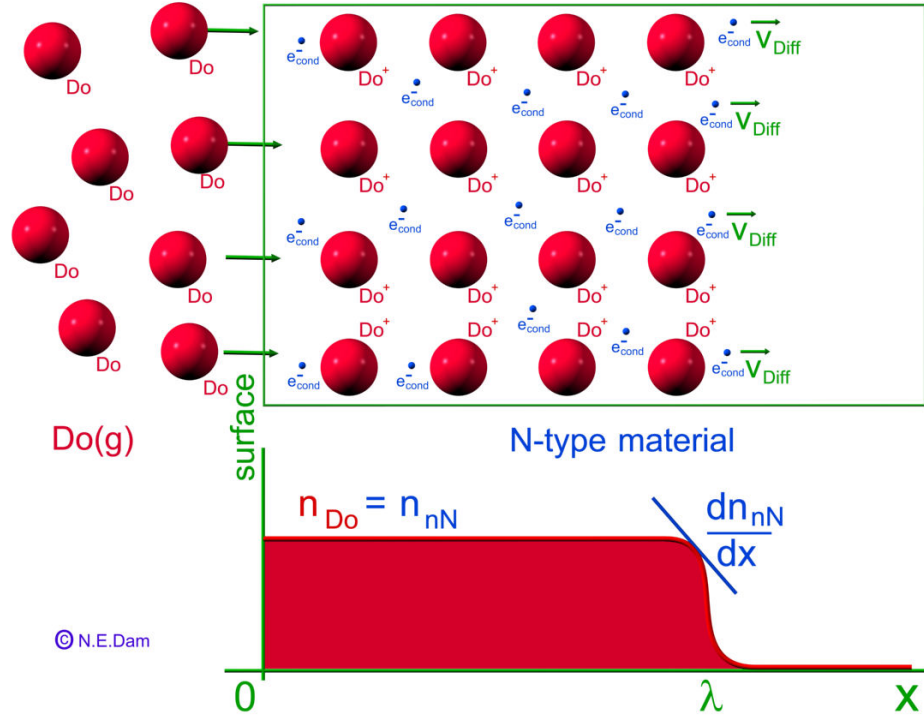


Figure 9.9. A diffusion current of electrons in a N-type semiconductor, where there is a variation of the concentration of donor atoms and electrons. The Si atoms are not shown.

9.4 Drift current and diffusion current

The current density in equation 9.20 on page 15 is in fact the so-called *drift current*

$$J_{Drift} = (n_{pI} \mu_p + n_{nI} \mu_n) e E \quad (9.36)$$

The charged particles are moving in random directions if there is no E-field. When an E-field is applied, the charged particles are exposed to an electric force. This motion is almost entirely in random directions but superimposed a slight drift in the direction of the electric force. This is explained in detail in chapter 8.

However, there is another kind of current, the so-called *diffusion current*. This current does not depend on an electric force. The diffusion current is due to a variation in the concentration of holes or electrons through the semiconductor.

Figure 9.9 shows a semiconductor (the green box). Donor atoms from a gas phase outside the semiconductor diffuse through the surface and into the N-type semiconductor,

where they become positively charged donor ions. E.g. it could be the P^+ ions in figure 9.6 on page 16. At the same time one electron is released from each donor atom. This electron is - almost certainly - excited to the conduction band. There are almost as many conduction electrons as there are donor atoms as stated in equation 9.23 on page 17.

After some time the diffusion has reached a certain depth λ . At this depth the concentration of donor atoms and the concentration of conduction electrons drops down to zero as shown in figure 9.9. We now remove the gas phase on the left side and cool down the semiconductor to room temperature, $T = 300$ K. The curve shows $n_{Do}(x)$ or $n_{nN}(x)$, where x is the depth from the surface. The so-called *concentration gradient* is given by dn_{nN}/dx , which is the slope of the curve tangent. It is zero, where $n_{nN}(x)$ is constant. However, at $x \simeq \lambda$ the tangent slope becomes very negative. In this region the electrons will try to diffuse to the right towards the right end of the semiconductor, where $n_{nN}(x) \simeq 0$. The charged particles diffuse from a region with a high concentration to a region with a low concentration. The formula for the diffusion current density is

$$J_{Diff} = (D_n \frac{dn_n}{dx} - D_p \frac{dn_p}{dx})e \quad (9.37)$$

D_n and D_p are the so-called *diffusion coefficients*. Some values are given below.

Element	μ_n	μ_p	Unit	D_n	D_p	Unit
Ge	0,390	0,190	$m^2/(Vs)$	101	49	$10^{-4}m^2/s$
Si	0,135	0,048	$m^2/(Vs)$	35	12	$10^{-4}m^2/s$

Table 9.2. Diffusion coefficient data for Ge and Si at $T = 300$ K.

The diffusion coefficients D_n and D_p are proportional with the mobilities μ_n and μ_p . This is expressed by the so-called *Einstein relations*:

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_B T}{e} = 26 \text{ mV at } T = 300 \text{ K} \quad (9.38)$$

The total current density is the sum of the two kinds of current densities

$$J = J_{Drift} + J_{Diff} \quad (9.39)$$

The values of the two terms on the right side of equation 9.39 are given by equation 9.36 and equation 9.37. If we decide to have the x -axis to the right, then J_{Drift} will be positive if the E-field is to the right giving a flow of positive charge to the right (or negative charge to the left). J_{Drift} will be negative if the E-field is to the left giving a flow of positive charge to the left (or negative charge to the right). Likewise for J_{Diff} . Please note that J_{Drift} and J_{Diff} may have opposite signs. If the sum of the two current densities is positive, then the total current density is to the right (flow of positive charge to the right).

Example

In figure 9.9 the tangent slope dn_{nN}/dx is negative at $x \simeq \lambda$. With this information, the first term in equation 9.37 becomes negative. The second term in equation 9.37 is vanishing because n_{pN} is vanishing in an N-type semiconductor, cf. equation 9.28 on page 17. With two negative terms, the right side of equation 9.37 becomes negative. This is equivalent with a positive current to the left in opposite direction of the x axis. In fact negative conduction electrons are diffusing to the right. If we connect the left end surface of the semiconductor to the positive pole of a battery and the right end surface to the negative pole, we get an E-field to the right and a positive value of E in 9.36. The conduction electrons are drifting to the left, which is equivalent with a positive drift current flow to the right. If J_{Drift} is dominating over J_{Diff} , the sum in 9.39 is positive, which means that the total flow of positive charge is to the right. However, the diffusion current counteracts the drift current to some extent.

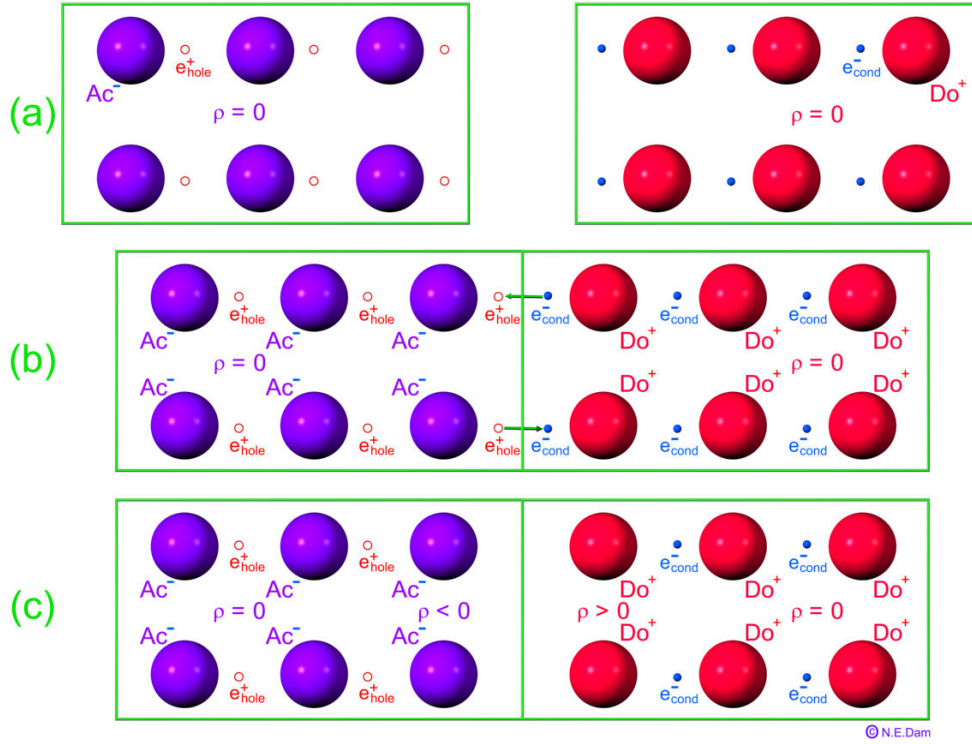


Figure 9.10. The simple PN-junction with the P-type semiconductor in the left side and the N-type semiconductor in the right side.

9.5 The PN-junction without a current

This section - and the subsequent many sections - is about a PN junction which is not connected to an external voltage source. The PN junction is not connected to any electrical circuit.

Figure 9.10 shows a P-type semiconductor in the left side and an N-type semiconductor in the right side. The Si atoms are not shown, only the doped ionized atoms, which are acceptor ions in the left side and donor ions in the right side. Of course, we also see positively charged holes in the P-type semiconductor in the left side and negatively charged conduction electrons in the N-type semiconductor in the right side. In figure (a) the P and N-type semiconductors are shown separated. In figure (b) the P and N-type semiconductors are merged together and a so-called PN-junction is formed. Close to the border between the P and N-type semiconductors we observe the following two phenomenons:

1. Holes from the left side are crossing the border and entering the N-type semiconductor in the right side. Here, the holes are quickly filled and neutralized with electrons.

2. Electrons from the right side are crossing the border and entering the P-type semiconductor in the left side. Here, the electrons are quickly neutralized when they hit some of the holes in the P-type semiconductor.

The two reactions above result in a narrow zone around the border, a so-called *depletion zone*, where holes and electrons have recombined and neutralized each other. In this zone, we have almost no holes and no conduction electrons as shown in figure 9.10(c) and figure 9.11(a). Hereby, we get a negatively charged “wall” of unshielded acceptor Ac^- ions to the left of the border and a positively charged “wall” of unshielded donor Do^+ ions to the right of the border as shown in figure 9.11(b). The blue volume to the left of the border has a negative volume density of charge, $\rho < 0$. The red volume to the right of the border has a positive volume density of charge, $\rho > 0$. The white volume to the left of the blue volume has a zero volume density of charge, $\rho = 0$, because there are almost as many holes as there are Ac^- ions. The white volume to the right of the red volume has a zero volume density of charge, $\rho = 0$, because there are almost as many conduction electrons as there are Do^+ ions.

Figure 9.11(b) also shows, that an E-field arises. The E-field is directed from the positively charged volume (red) to the negatively charged volume (blue). If we have a particle with a charge q_0 in an E-field (the E-field is generated by other charged particles than q_0), there will be an electric force on the particle with the charge q_0 and this force is proportional with the E-field. The E-field is defined as the electric force per charge:

$$\vec{E} \equiv \frac{\vec{F}_e}{q_0} \quad (9.40)$$

Thereby, the electric force acting on q_0 may be calculated from the E-field

$$\vec{F}_e = q_0 \vec{E} \quad (9.41)$$

Let us consider figure 9.11(b) again. If there should be a hole left in the blue volume to the left of the border, this hole is positively charged, $q_0 = e$. Hence, according to equation 9.41, there will be an electric force vector in the same direction as the E-field vector. This force is directed towards left and the force will stop further migration of holes across the border to the N-type material.

If there should be a conduction electron left in the red volume to the right of the border, this electron is negatively charged, $q_0 = -e$. Hence, according to equation 9.41, there will be an electric force vector in the opposite direction of the E-field vector. This force is directed towards right and the force will stop further migration of electrons across the border to the P-type material. To summarize the above part of section 9.5:

When the P and N-type semiconductors are merged together, there will be a migration of electrons and holes across the border. The electrons are absorbed by holes on the other side of the border and the holes are recombined with electrons on the opposite side of the

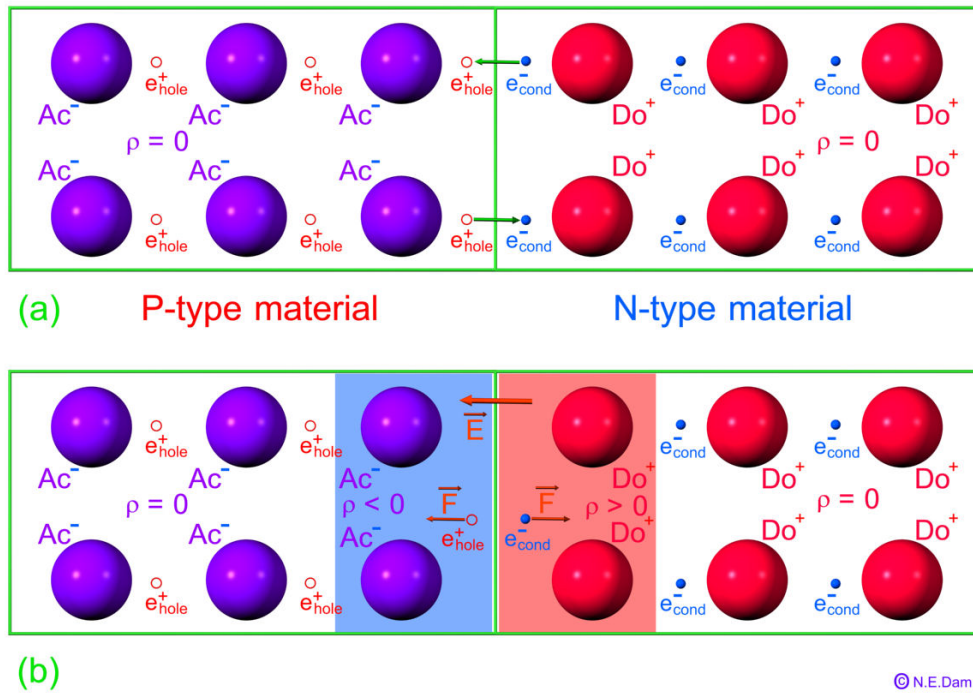


Figure 9.11. PN junction with positive volume charge density on one side of the boundary and negative volume charge density on the other side of the boundary.

border. The result is a neutralized region, a depletion zone with an E-field. The electric forces now stop further migration.

Figure 9.11(a) also shows that every time a hole leaves the P-type material and migrate into the N-type material, it creates a single Ac^- ion without a hole. Soon after the migrated hole arrives on the right side of the border and combines with one electron. It creates a single Do^+ ion without the electron, that originally was in excess (e.g. the fifth electron coming with the phosphorus (P) atom shown in figure 9.6 on page 16. The result is, that migration of one hole from left to right creates a single Ac^- ion without a hole and a single Do^+ ion without an “excess “ electron.

Likewise, when a conduction electron migrates from right to left across the border, it creates a single Do^+ ion without an “excess “ electron and a single Ac^- ion without a hole.

In conclusion, there are created as many Do^+ ions without an “excess“ electron as there are created Ac^- ions without a hole. The number of Do^+ ions without an “excess“ electron in the red volumen equals the number of Ac^- ions without a hole in the blue volumen.

Now, let us consider figure 9.12.

- (a) shows, that there may be more layers of Ac^- ions in the negatively charged blue volumen to the left of the border, not only a single layer with two atoms as shown in the very simplified figure 9.11! Likewise for the red volumen. In this figure we see, that the number of Do^+ ions without an “excess“ electron in the red volumen equals the number of Ac^- ions without a hole in the blue volumen. In figure (a) we have the same density of atoms and the same degree of doping on both sides of the border, i.e. the blue and the red regions occupy the same volume.
- (b) In this figure we have the same density of host atoms (which are not shown) but not the same degree of doping on both sides of the border. The degree of doping on the right side is only half of the degree of doping on the left side. We remember, that the number of Do^+ ions without an “excess“ electron in the red volumen equals the number of Ac^- ions without a hole in the blue volumen. Hence, the red region (with the Do^+ ions without an “excess“ electron) occupies twice the volume relative to the blue volumen.

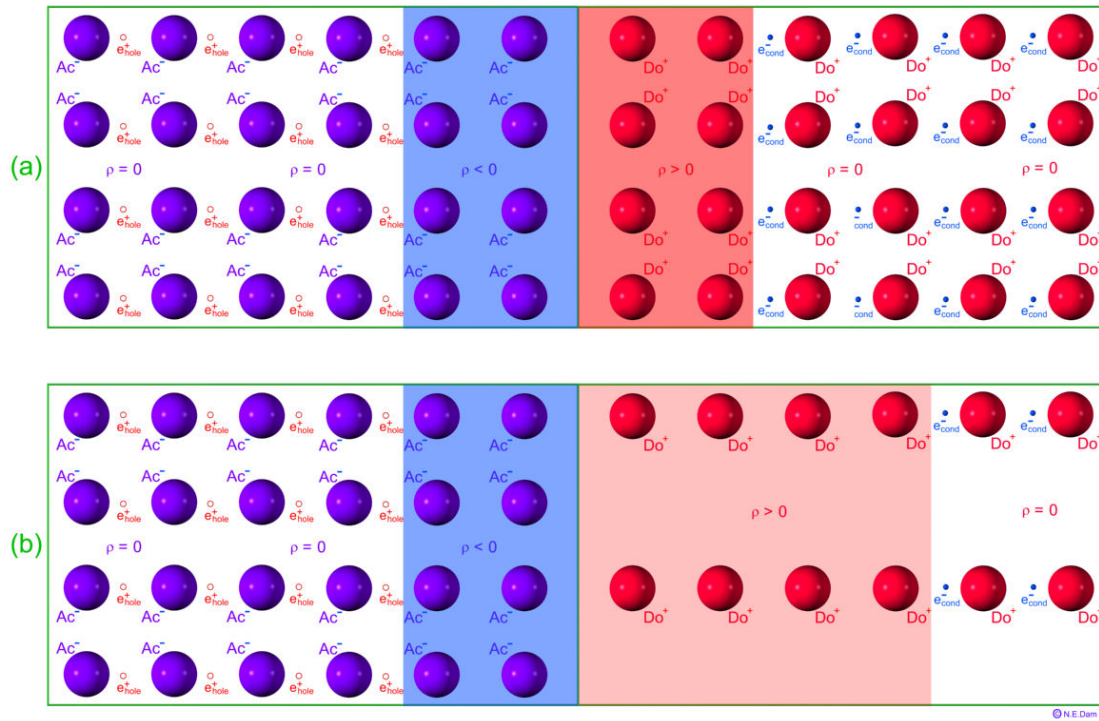


Figure 9.12. PN junction with different penetration depths for the regions of positive volume charge density and negative volume charge density on either side of the boundary, ©N.E.Dam.

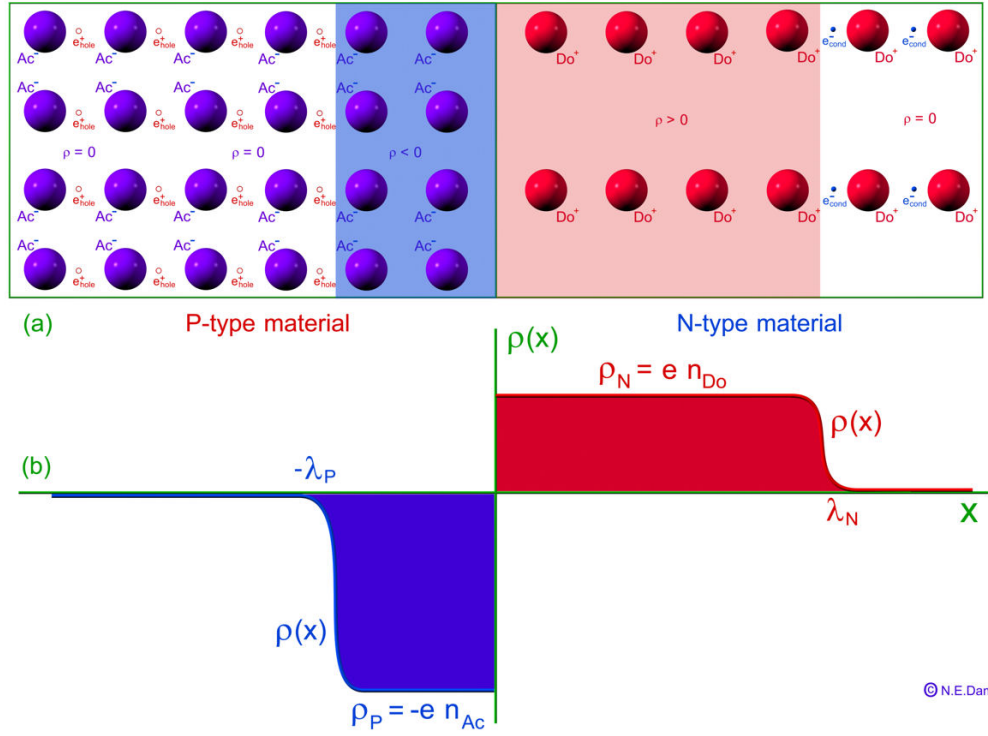


Figure 9.13. The PN-junction and the volume charge density.

9.6 Profile of the volume charge density

In this section we consider the profile of the volume charge density through the PN junction. Figure 9.13 shows the volume charge density ρ as a function of the x coordinate. The x -axis is here perpendicular to the boundary between the P and N area with the zero point in the boundary itself.

On the left side of the blue region in the P-type material there are almost equally many holes and Ac^- ions, therefore we have

$$\rho(x) = 0, \quad x < -\lambda_P \quad (9.42)$$

Here λ_P is the thickness of the blue area, i.e. the penetration depth in the P area.

On the right side of the red region in the N-type material there are almost equally many conduction electrons and Do^+ ions, therefore we have

$$\rho(x) = 0, \quad \lambda_N < x \quad (9.43)$$

Here λ_N is the thickness of the red area, i.e. the penetration depth in the N area.

Inside the blue region in the P-type material the number of Ac^- ions per m^3 is n_{Ac} . Each Ac^- ion has the charge $-e$. Hence, the charge per m^3 is $-e n_{Ac}$. Therefore we have

$$\rho(x) = \rho_P = -e n_{Ac}, \quad -\lambda_P < x < 0 \quad (9.44)$$

Inside the red region in the N-type material the number of Do^+ ions per m^3 is n_{Do} . Each Do^+ ion has the charge e . Hence, the charge per m^3 is $e n_{Do}$. Therefore we have

$$\rho(x) = \rho_N = e n_{Do}, \quad 0 < x < \lambda_N \quad (9.45)$$

The volume of the blue region is $\lambda_P A$, where A is the cross section area perpendicular to the x-axis. The amount of charge per m^3 is $-e n_{Ac}$. Hence, the total amount of charge in the blue region is: $-e n_{Ac} \lambda_P A$.

The volume of the red region is $\lambda_N A$. The amount of charge per m^3 is $e n_{Do}$. Hence, the total amount of charge in the red region is: $e n_{Do} \lambda_N A$.

As explained above, the number of Ac^- ions in the blue region equals the number of Do^+ ions in the red region. Therefore, the absolute amount of charge in the blue region equals the amount of charge in the red region:

$$e n_{Ac} \lambda_P A = e n_{Do} \lambda_N A \Rightarrow e n_{Ac} \lambda_P = e n_{Do} \lambda_N \Rightarrow -\rho_P \lambda_P = \rho_N \lambda_N \quad (9.46)$$

We return to the profile of the volume charge density $\rho(x)$ in figure 9.13b. We see, that the left side of equation 9.46 equals the blue area and the right side of equation 9.46 equals the red area in the same profile.

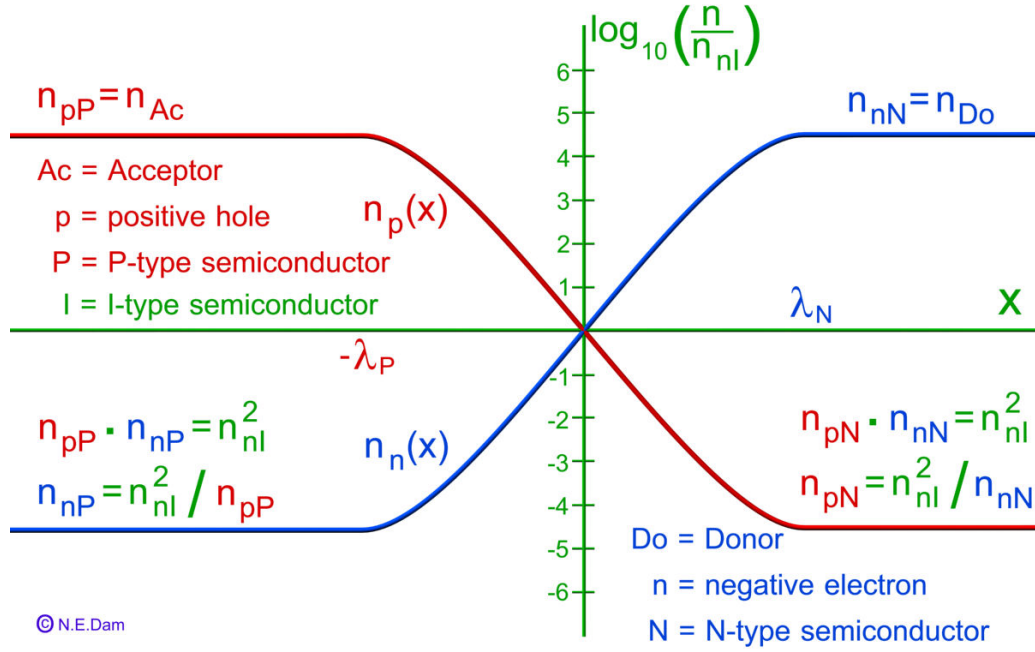


Figure 9.14. Concentration profiles for a PN-junction with the same degree of doping on both sides.

9.7 Concentration profiles

Figure 9.14 shows the concentration profile $n(x)$ through the PN-junction. As defined earlier, the concentration n is the number of particles per m^3 . This figure corresponds to the same degree of doping on both sides of the border as in figure 9.12a on page 28.

The red curve in figure 9.14 is the concentration profile $n_p(x)$ of the positively charged holes from the P-type material (P-type semiconductor) through the PN-junction to the N-type material (N-type semiconductor).

In the P-type material the concentration of holes is almost the same as the concentration of acceptor ions as presented in equation 9.30 on page 19. In the P-type material the concentration of conduction electrons is determined using equation 9.12 on page 13 which is also valid in the P-type material

$$n_{pP} \cdot n_{nP} = n_{nI}^2 \Rightarrow n_{nP} = \frac{n_{nI}^2}{n_{pP}} \simeq \frac{n_{nI}^2}{n_{Ac}} \quad (9.47)$$

Example: How to calculate the concentration profiles of conduction electrons and holes across the border in the PN-junction

If we consider figure 9.14, we will be able to sketch the concentration profiles of conduction electrons (blue curves) and holes (red curves) if we calculate n_{pP} , n_{nP} , n_{nN} , and n_{pN} . We use data from table 9.1 on page 15 for a Si semiconductor and assume that:

- The concentration of host atoms is $n_{Ho} = 5 \cdot 10^{28} \text{atoms}/m^3$
- The degree of doping in the P-type semiconductor is $\zeta_P = \frac{1}{5} \cdot 10^{-6}$
- The degree of doping in the N-type semiconductor is $\zeta_N = \frac{1}{5} \cdot 10^{-9}$. This value is perhaps not very realistic, but it simplifies this example and it makes it easier to see the difference between the two sides of the PN-transition.
- The real concentration of thermal excited conduction electrons in an I-type semiconductor is $n_{nI} = 1,45 \cdot 10^{16} \text{electrons}/m^3$. In order to simplify this example we use $n_{nI} = 10^{16} \text{electrons}/m^3$.

Now, we find the following

- *The concentration of holes in the P-type semiconductor* is calculated this way:
 $n_{pP} \simeq n_{Ac} = \zeta_P \cdot n_{Ho} = \frac{1}{5} \cdot 10^{-6} \cdot 5 \cdot 10^{28} \text{holes}/m^3 = 10^{22} \text{holes}/m^3$.
 We also calculate this concentration relative to n_{nI} this way: $\frac{n_{pP}}{n_{nI}} = \frac{10^{22}}{10^{16}} = 10^6$
- *The concentration of conduction electrons in the P-type semiconductor* is calculated using equation 9.47 this way: $n_{nP} = \frac{n_{nI}^2}{n_{pP}} \simeq \frac{n_{nI}^2}{n_{Ac}} = \frac{(10^{16})^2}{10^{22}} = 10^{10} \text{electrons}/m^3$
 We also calculate this concentration relative to n_{nI} this way: $\frac{n_{nP}}{n_{nI}} = \frac{10^{10}}{10^{16}} = 10^{-6}$
- *The concentration of conduction electrons in the N-type semiconductor* is calculated this way:
 $n_{nN} \simeq n_{Do} = \zeta_N \cdot n_{Ho} = \frac{1}{5} \cdot 10^{-9} \cdot 5 \cdot 10^{28} \text{electrons}/m^3 = 10^{19} \text{electrons}/m^3$.
 We also calculate this concentration relative to n_{nI} this way: $\frac{n_{nN}}{n_{nI}} = \frac{10^{19}}{10^{16}} = 10^3$
- *The concentration of holes in the N-type semiconductor* is calculated using equation 9.47 this way: $n_{pN} = \frac{n_{nI}^2}{n_{nN}} \simeq \frac{n_{nI}^2}{n_{Do}} = \frac{(10^{16})^2}{10^{19}} = 10^{13} \text{holes}/m^3$
 We also calculate this concentration relative to n_{nI} this way: $\frac{n_{pN}}{n_{nI}} = \frac{10^{13}}{10^{16}} = 10^{-3}$

Now, we have data enough to sketch the two concentration profiles for $n_n(x)$ and $n_p(x)$. Figure 9.15 shows the concentration profiles through the PN-junction. This figure corresponds to different degrees of doping on the two sides of the border as in figure 9.13 on page 29.

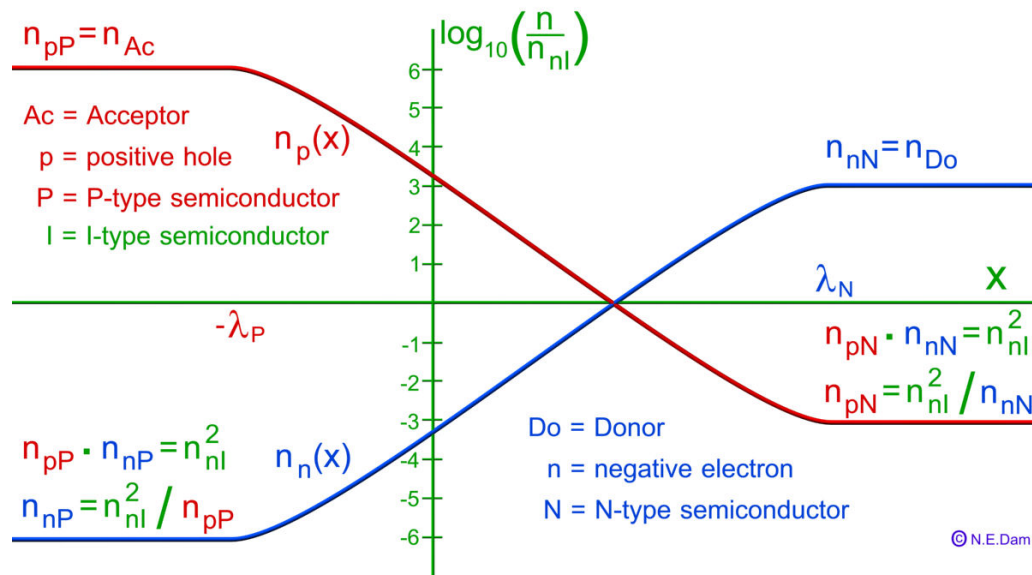


Figure 9.15. Concentration profiles for a PN-junction with different degree of doping in the P-type material and in the N-type material.

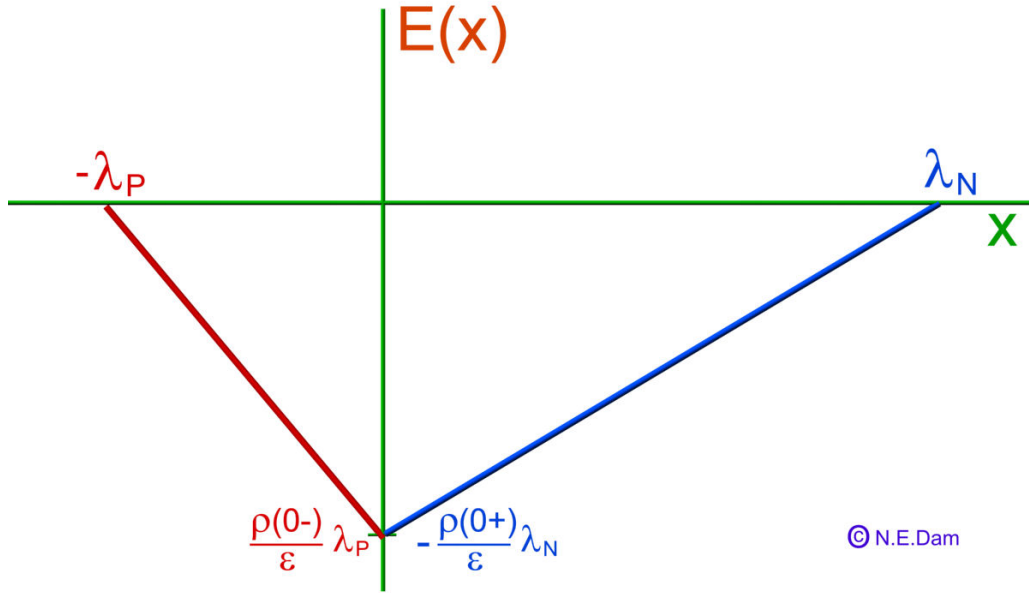


Figure 9.16. The E-field profile through the PN-junction.

9.8 E-field profile

Figure 9.16 shows the final result. It is immediately understandable that there is the largest E-field in the center at $x = 0$. In this place, there is the largest amount of negative charge on the left side and the largest amount of positive charge on the right side. This appears, for example, of figure 9.13 on page 29. In the remainder of this section, we will derive the E-field profile using Gauss's law in differential form. It is assumed that you are familiar with chapters 2, 3 and 6.

Gauss's law in differential form is

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \quad (9.48)$$

Here,

$\vec{E} = (E_x(x, y, z), E_y(x, y, z), E_z(x, y, z))$ is the E-field vector at a point (x, y, z) ,

ρ is the volume charge density at the same point (x, y, z)

ϵ is the absolute permittivity, which is a material constant.

Finally, the differential operator - the so-called Del operator - $\vec{\nabla}$ is defined by

$$\vec{\nabla} \equiv \left\{ \left(\frac{\partial}{\partial x} \right)_{y,z}, \left(\frac{\partial}{\partial y} \right)_{x,z}, \left(\frac{\partial}{\partial z} \right)_{x,y} \right\} \quad (9.49)$$

The divergence of $\vec{E}$ is explained in chapter 3. It is defined by

$$\vec{\nabla} \cdot \vec{E} \equiv \left\{ \left(\frac{\partial}{\partial x} \right)_{y,z}, \left(\frac{\partial}{\partial y} \right)_{x,z}, \left(\frac{\partial}{\partial z} \right)_{x,y} \right\} \cdot (E_x, E_y, E_z) = \left(\frac{\partial E_x}{\partial x} \right)_{y,z} + \left(\frac{\partial E_y}{\partial y} \right)_{x,z} + \left(\frac{\partial E_z}{\partial z} \right)_{x,y} \quad (9.50)$$

In this case the E-field is parallel with the x-axis, i.e.

$$(E_x, E_y, E_z) = (E, 0, 0) \quad (9.51)$$

NB! A negative value of E_x or E means that vector $\vec{E}$ points to the left.

Using Gauss's law we find

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon} \Rightarrow \left\{ \left(\frac{\partial}{\partial x} \right)_{y,z}, \left(\frac{\partial}{\partial y} \right)_{x,z}, \left(\frac{\partial}{\partial z} \right)_{x,y} \right\} \cdot (E, 0, 0) = \frac{dE}{dx} = \frac{\rho}{\epsilon} \Rightarrow dE = \frac{\rho}{\epsilon} dx \quad (9.52)$$

The integration of dE is split into two intervals:

1. interval, the P-type material where $x \in [-\lambda_P, 0]$

$$\int_{-\lambda_P}^x dE = \int_{-\lambda_P}^x \frac{\rho}{\epsilon} dx' = \frac{\rho}{\epsilon} [x']_{x'=-\lambda_P}^{x'=x} \Rightarrow E(x) - E(-\lambda_P) = \frac{\rho}{\epsilon} (x - (-\lambda_P)) \quad (9.53)$$

The two charged regions in the N-type material and in the P-type material act as a kind of double layer in a capacitor. The E-field outside such a double layer is almost zero as explained in chapter 3. Hence, E is almost zero for $x \leq -\lambda_P$ and for $\lambda_N \leq x$ meaning that

$$E(-\lambda_P) \simeq 0 \text{ and } E(\lambda_N) \simeq 0 \quad (9.54)$$

Hereby, we get

$$E(x) = \frac{\rho}{\epsilon} (x + \lambda_P) = \frac{\rho}{\epsilon} x + \frac{\rho}{\epsilon} \lambda_P, \quad x \in [-\lambda_P, 0] \quad (9.55)$$

This is a straight line with the slope $\frac{\rho}{\epsilon} < 0$. The volume charge density is considered to be constant negative with the value ρ_P in the interval $x \in [-\lambda_P, 0]$, cf. figure 9.13b on page 29. From equation 9.55 we see, that

$$x = -\lambda_P \Rightarrow E(x) = 0 \text{ and } x = 0 \Rightarrow E(x) = \frac{\rho}{\epsilon} \lambda_P \quad (9.56)$$

These two points determine the straight line as shown in figure 9.16, where $\rho(0-) = \rho_P < 0$ is the value of ρ from the left side at $x = 0$.

2. interval, the N-type material where $x \in [0, \lambda_N]$

$$\int_0^x dE = \int_0^x \frac{\rho}{\epsilon} dx' = \frac{\rho}{\epsilon} [x']_{x'=0}^{x'=x} \Rightarrow E(x) - E(0) = \frac{\rho}{\epsilon}(x - (0)) \quad (9.57)$$

If we let $x = \lambda_N$ in equation 9.57 and use equation 9.54, we find

$$E(\lambda_N) - E(0) = \frac{\rho}{\epsilon}\lambda_N \Rightarrow E(0) = -\frac{\rho}{\epsilon}\lambda_N \quad (9.58)$$

This value of $E(0)$ is substituted into equation 9.57 and we find

$$E(x) - E(0) = E(x) - (-\frac{\rho}{\epsilon}\lambda_N) = \frac{\rho}{\epsilon}x \Rightarrow E(x) = \frac{\rho}{\epsilon}x - \frac{\rho}{\epsilon}\lambda_N \quad (9.59)$$

This is a straight line with the slope $\frac{\rho}{\epsilon} > 0$. The volume charge density is considered to be constant positive with the value ρ_N in the interval $x \in [0, \lambda_N]$, cf. figure 9.13b on page 29. From equation 9.59 we see, that

$$x = \lambda_N \Rightarrow E(\lambda_N) = 0 \text{ and } x = 0 \Rightarrow E(0) = -\frac{\rho}{\epsilon}\lambda_N \quad (9.60)$$

These two points determine the straight line as shown in figure 9.16, where $\rho(0+) = \rho_N > 0$ is the value of ρ from the right side at $x = 0$.

We note that the E-field profile on the left side of $x = 0$ and the E-field profile on the right side of $x = 0$ meet at the same point at $x = 0$. We therefore have one - and only one - value for E-field at the point $x = 0$.

- The E-field from the left side at $x = 0$ is denoted $E(0-)$.
- The E field from the right side at $x = 0$ is denoted $E(0+)$.

We thus have:

$$E(0-) = \frac{\rho_P}{\epsilon}\lambda_P = E(0+) = -\frac{\rho_N}{\epsilon}\lambda_N \quad (9.61)$$

this implies that

$$\rho_P\lambda_P = -\rho_N\lambda_N \quad (9.62)$$

This is identical to equation 9.46 on page 30 which expresses that the amount of charge to the left of $x = 0$ is equal to minus the amount of charge to the right of $x = 0$. Note, that equation 9.61 allows us to determine the maximum E-field magnitude at $x = 0$. This E-field value may be extremely high.

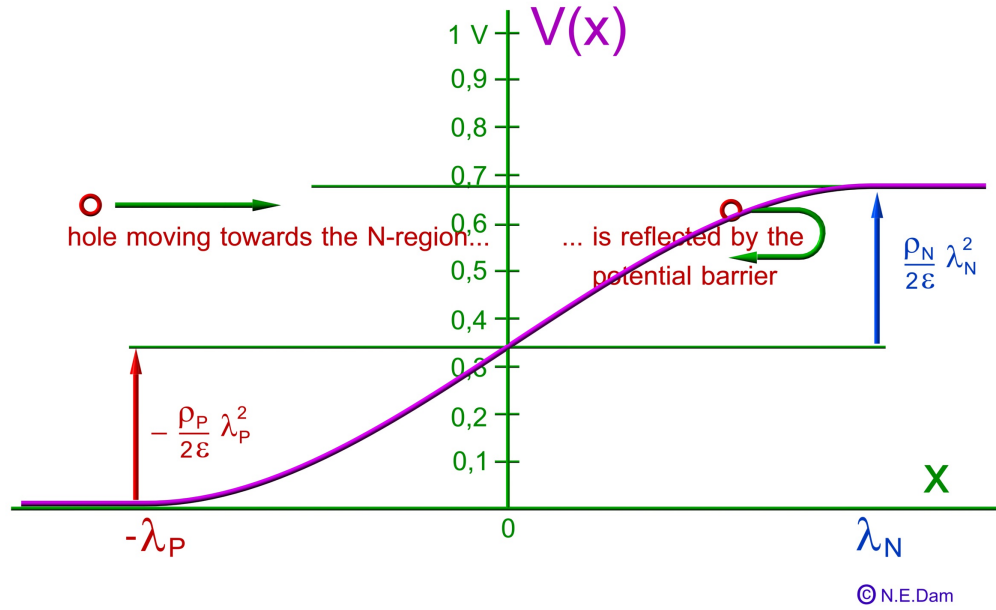


Figure 9.17. The potential profile through the PN-junction.

9.9 Potential profile

The purpose of finding the E-field profile in the previous section was in reality to be able to determine the potential profile. The final result for the potential profile is shown in figure 9.17. The actual derivation of the potential profile is divided into two intervals in the same way as when determining the E-field profile. But this time we integrate $E(x)$.

From chapter 5 we have the following integral for the potential drop from a to b:

$$V_{ab} = \int_a^b \vec{E} \cdot d\vec{l} = V(a) - V(b) \quad (9.63)$$

Hence, the increase in potential is:

$$\Delta V = V_{ba} = \int_b^a \vec{E} \cdot d\vec{l} = V(b) - V(a) = -V_{ab} = - \int_a^b \vec{E} \cdot d\vec{l} \quad (9.64)$$

In this case we have $\vec{E} = (E_x, E_y, E_z) = (E, 0, 0)$. Therefore we find

$$\Delta V = - \int_a^b \vec{E} \cdot d\vec{l} = - \int_a^b (E, 0, 0) \cdot (dx, dy, dz) = - \int_a^b E dx \quad (9.65)$$

In the following we define the zero level for the potential to be at the P-type material and we calculate the increase in potential, ΔV , from this level. Note that x' in the following context does not denote the differential coefficient of x , but simply the variable x in the integral to distinguish the variable x , which we denote by x' , from the upper limit of the integral, which is x . The integration of E is split into two intervals:

1. interval, the P-type material where $x \in [-\lambda_P, 0]$

In this interval we have found the following expression for the E-field, cf. equation 9.55

$$E(x) = \frac{\rho_P}{\epsilon}(x + \lambda_P), x \in [-\lambda_P, 0] \quad (9.66)$$

This expression is used in the following integral

$$\Delta V = - \int_{-\lambda_P}^x E dx' = - \int_{-\lambda_P}^x \frac{\rho_P}{\epsilon}(x' + \lambda_P) dx' \Rightarrow \quad (9.67)$$

$$\Delta V = - \frac{\rho_P}{2\epsilon} [(x' + \lambda_P)^2]_{x'=-\lambda_P}^{x'=x} = - \frac{\rho_P}{2\epsilon} [(x + \lambda_P)^2 - 0^2] = - \frac{\rho_P}{2\epsilon} (x + \lambda_P)^2 \quad (9.68)$$

As mentioned above we define the zero level for the potential to be at the P-type material meaning that $V(x) = 0$ when $x < -\lambda_P$. Furthermore ΔV is the increase in the potential from $-\lambda_P$ to x . Therefore we have that $V(x) = \Delta V$ when $x \in [-\lambda_P, 0]$. The final result for $V(x)$ in the P-type material becomes:

$$V(x) = 0, x < -\lambda_P \text{ and } V(x) = - \frac{\rho_P}{2\epsilon} (x + \lambda_P)^2, x \in [-\lambda_P, 0] \quad (9.69)$$

From equation 9.69 we see, that

$$x = -\lambda_P \Rightarrow V(-\lambda_P) = 0 \text{ and } x = 0 \Rightarrow V(0) = - \frac{\rho_P}{2\epsilon} \lambda_P^2 \quad (9.70)$$

The result in equation 9.69 expresses that $V(x)$ in the interval $[-\lambda_P, 0]$ is part of a convex parabola³ with it's minimum at $x = -\lambda_P$.

2. interval, the N-type material where $x \in [0, \lambda_N]$

In this interval we have found the following expression for the E-field, cf. equation 9.59

$$E(x) = \frac{\rho_N}{\epsilon}(x - \lambda_N), x \in [0, \lambda_N] \quad (9.71)$$

³The function is convex when the line segment between any two distinct points lies above the graph between the two points. The second derivative is positive.

This expression is used in the following integral, where we find the increase in potential V from 0 to x

$$\Delta V = V(x) - V(0) = - \int_0^x E dx' = - \int_0^x \frac{\rho_N}{\epsilon} (x' - \lambda_N) dx' \Rightarrow \quad (9.72)$$

$$\Delta V = - \frac{\rho_N}{2\epsilon} [(x' - \lambda_N)^2]_{x'=0}^{x'=x} = - \frac{\rho_N}{2\epsilon} [(x - \lambda_N)^2 - (-\lambda_N)^2] = - \frac{\rho_N}{2\epsilon} (x^2 - 2\lambda_N x) \quad (9.73)$$

This is the increase ΔV in potential from 0 to x . To find the final function $V(x)$ where $x \in [0, \lambda_N]$, we must add the value of the potential at $x = 0$, cf. equation 9.70. The final result for $V(x)$ in the N-type material becomes:

$$V(x) = V(0) + \Delta V = - \frac{\rho_P}{2\epsilon} \lambda_P^2 - \frac{\rho_N}{2\epsilon} (x^2 - 2\lambda_N x) = - \frac{\rho_P}{2\epsilon} x^2 + \frac{\rho_N}{2\epsilon} 2\lambda_N x - \frac{\rho_P}{2\epsilon} \lambda_P^2 \quad (9.74)$$

From equation 9.74 we see, that

$$x = 0 \Rightarrow V(0) = - \frac{\rho_P}{2\epsilon} \lambda_P^2 \text{ and } x = \lambda_N \Rightarrow V(\lambda_N) = - \frac{\rho_P}{2\epsilon} \lambda_P^2 + \frac{\rho_N}{2\epsilon} \lambda_N^2 \quad (9.75)$$

The result in equation 9.74 expresses that $V(x)$ in the interval $[0, \lambda_N]$ is part of a concave parabola⁴ with it's maximum at $x = \lambda_N$.

Since the E field is zero to the right of $x = \lambda_N$, then $\Delta V = 0$ to the right of λ_N , i.e. the potential is constant to the right of λ_N . The final result for $V(x)$ in the N-type material becomes:

$$V(x) = - \frac{\rho_P}{2\epsilon} \lambda_P^2 - \frac{\rho_N}{2\epsilon} (x^2 - 2\lambda_N x), \quad x \in [0, \lambda_N] \quad (9.76)$$

and

$$V(x) = - \frac{\rho_P}{2\epsilon} \lambda_P^2 + \frac{\rho_N}{2\epsilon} \lambda_N^2, \quad \lambda_N \leq x \quad (9.77)$$

The final result for the potential profile is shown in figure 9.17 at the beginning of this section. In the P-type material we have a plateau and part of a convex parabola given by equation 9.69. In the N-type material we have a part of a concave parabola and a plateau given by equation 9.76 respectively equation 9.77.

The latter plateau is the total height of the potential barrier, also known as the built-in potential:

$$V_{BI} = V(\lambda_N) = - \frac{\rho_P}{2\epsilon} \lambda_P^2 + \frac{\rho_N}{2\epsilon} \lambda_N^2 \quad (9.78)$$

This is the real reason why a Si diode must be biased with a little under 0.7 V in the conduction direction in order to get current through the diode.

⁴The function is concave when the line segment between any two distinct points lies below the graph between the two points. The second derivative is negative.

List of Figures

9.1	A unit cell with the so-called diamond structure. This is the crystallographic structure of carbon (C), silicon (Si) and germanium (Ge).	7
9.2	A so-called tetrahedron with 5 Si atoms is a part of the unit cell with the diamond structure shown in figure 9.1.	8
9.3	A simplified drawing of the Si crystal structure.	9
9.4	A pure (intrinsic) Si semiconductor, where one of the covalent electrons is excited to the conduction band.	10
9.5	The grey material is supposed to be an intrinsic semiconductor. The electrons are moving in the opposite direction of the E-field, the holes are moving in the same direction as the E-field.	12
9.6	An N-type semiconductor.	16
9.7	A P-type semiconductor where pure Si has been doped by B.	18
9.8	A P-type semiconductor. The covalent bond with the missing electron at the B atom becomes a complete covalent bond with 2 electrons because an electron is transferred from another covalent bond in the neighbourhood. . .	20
9.9	A diffusion current of electrons in a N-type semiconductor, where there is a variation of the concentration of donor atoms and electrons. The Si atoms are not shown.	21
9.10	The simple PN-junction with the P-type semiconductor in the left side and the N-type semiconductor in the right side.	24
9.11	PN junction with positive volume charge density on one side of the boundary and negative volume charge density on the other side of the boundary. . . .	26
9.12	PN junction with different penetration depths for the regions of positive volume charge density and negative volume charge density on either side of the boundary, ©N.E.Dam.	28
9.13	The PN-junction and the volume charge density.	29
9.14	Concentration profiles for a PN-junction with the same degree of doping on both sides.	31
9.15	Concentration profiles for a PN-junction with different degree of doping in the P-type material and in the N-type material.	33

9.16 The E-field profile through the PN-junction.	34
9.17 The potential profile through the PN-junction.	37

List of Tables

9.1	Hole and electron mobility data for Ge and Si at $T = 300$ K.	15
9.2	Diffusion coefficient data for Ge and Si at $T = 300$ K.	22

Glossary

antimony grundstoffet antimon som betegnes Sb efter latin Stibium. 17

arsenic grundstoffet arsen. NB! Arsenik er giftstoffet arsentrioxid As_2O_3 . 17

atomic crystal lattice atomare krystal gitter. 9

concave parabola konkav parabel er en parabel, som er ”nedad hul”, dvs. den anden ordens afledede er negativ. Dette er tilfældet når koefficienten til anden ordens leddet med x^2 er negativ. 39

convex parabola konveks parabel er en parabel, som er ”opad hul”, dvs. den anden ordens afledede er positiv. Dette er tilfældet når koefficienten til anden ordens leddet med x^2 er positiv. 38

deficit of mangel på. 9

excited to the conduction band eksiteret til ledningsbåndet, dvs. elektronen løftes op i et højt liggende energiniveau i det såkaldte ledningsbånd, som er nogle meget tætliggende energiniveauer, hvor elektronerne kan surfe igennem hele krystalstrukturen og altså lede elektrisk strøm, deraf navnet ledningsbånd. 9

PN-junction Direkte oversat er ”junction” et knudepunkt. I denne sammenhæng er der tale om en grænseflade eller en overgang (eng.”transition”) mellem et P-område med positive ladningsbærere (huller) og et N-område med negative ladningsbærere (elektroner). På dansk kan man også bruge udtrykket PN-overgang. 24

Nomenclature

ϵ	ϵ is the absolute permittivity.
λ_N	Penetration depth in the N area.
λ_P	Penetration depth in the P area.
ρ	$\rho(x,y,z)$ is the volume charge density at point (x,y,z) .
$\vec{\nabla}$	The Del operator defined by $\vec{\nabla} \equiv \{(\frac{\partial}{\partial x})_{y,z}, (\frac{\partial}{\partial y})_{x,z}, (\frac{\partial}{\partial z})_{x,y}\}$.
$\vec{E}$	$\vec{E} = (E_x(x,y,z), E_y(x,y,z), E_z(x,y,z))$ is the E-field vector at a point (x,y,z) .
n_{Ac}	The concentration of acceptor atoms (number of acceptor atoms per m^3).
n_{Do}	The concentration of donor atoms (number of donor atoms per m^3).
n_{Ho}	The concentration of host atoms.
n_{HI}	The concentration of host atoms.
n_{nI}	The number of negatively charged conduction electrons per m^3 of the Intrinsic material. Note that n in the subscript is used for negatively charged conduction electrons and subscript I means Intrinsic semiconductor.
n_{pI}	The number of positively charged holes per m^3 of the Intrinsic material. Note that p in the subscript is used for positively charged holes and subscript I means Intrinsic semiconductor.
U_g	The energy it takes to excite an electron from a covalent bond to the conduction band.
v_d	Drift velocity in general.
v_n	Drift velocity of negatively charged conduction electrons moving in the opposite direction of the E-field.
v_p	Drift velocity of positively charged holes moving in the same direction as the E-field.

Index

- Absolute permittivity, 35
- Acceptor atom, 18
- Built-in potential, 39
- Concentration gradient, 22
- Concentration, conduction electrons, 11
- Concentration, holes, 11
- Coordination number, 8
- Covalent bonds, 8
- Covalent electron, 8
- Crystal structure of Si and Ge, 7
- Current density and conductivity, 14
- Current density, definition, 13
- Current density, holes and electrons, 15
- Current density, thermal ex. electrons, 14
- Current density, thermal excited holes, 14
- Degree of doping, 16, 18
- Del operator, 35
- Depletion zone, 25
- Diamond structure, 7
- Diffusion coefficients, 22
- Diffusion current, 21
- Donor atom, 16
- Doping, 16, 18
- Drift current, 21
- Drift current and diffusion current, 21
- Drift velocities, holes and electrons, 11
- E-field, 35
- Einstein relations, 22
- Electric current, definition, 12
- Electric force and electric E-field, 11
- Energy gap, 10, 14
- Gauss's law in differential form, 34
- Hole, 9
- Host, 16, 18
- I-type semiconductor, 7
- Intrinsic semiconductors, 7
- Mobility of electrons, 14
- Mobility of holes, 14
- N-type semiconductor, 16
- Ohm's law expressed by $\vec{E}$ and $\vec{J}$, 13
- P-type semiconductor, 18
- Penetration depth in the N area, 29
- Penetration depth in the P area, 29
- PN-junction without a current, 24
- PN-junction, charge density profile, 29
- PN-junction, concentration profiles, 31
- PN-junction, E-field profile, 34
- PN-junction, potential profile, 37
- Thermal excited electrons and holes, 9
- Volume charge density, 35